Relation: subset of a Cartesian Product

A = courses at UCF

B = set of instructors at UCF

R subset A x B, as follows:

{(x, y) | course x is taught by instructor y}

R = { (COT3100, Guha), (COP4516, Guha), (COP3502, Meade), (COT3100, Gerber), (COP3502, Szumlanski), …}

Number of possible relations over A x B = $2^{|A||B|}$

Focus on relations over the set A x A.

Properties we can investigate:

A = {1, 2, 3, 4}

Reflexive: For all a in the set A, (a, a) is an element of R

R = {(1,1), (1,2), (1,3), (2,2), (2,4), (3, 3), (4, 4)} is reflexive

S = {(1,1), (1,2), (2, 2), (2,3), (3,4), (4,1)} is not reflexive

T = {(1,2), (2,3), (3,4), (4,1)} is not reflexive

Irreflexive: For all a in the set A, (a, a) is NOT element of R

R and S are NOT irreflexive, T is irreflexive.

Symmetric: For all a and b in set A, if (a, b) is a member of R, then (b, a) is also a member of R.

Anti-symmetric: For all a and b in set A, if (a, b) is a member of R AND (b, a) is also a member of R, then a = b.

For all a, b in A, If a != b and (a,b) is in R, then (b,a) is NOT in R.

Transitive: for all a,b,c in A, if (a,b) is in R and (b, c) is in R, then (a,c) is in R.

R subset of A x B

S subset of B x C

Define S ° R = {(a, c) | there exists b such that (a,b) is in R and (b,c) is in S}

Reminder of function composition g(f(x)) means, plug in x to f and get its output. Then, take that value and input it into g…

r(R) = reflexive closure it’s the set you get when you add all the elements you need to to make R reflexive

= R union {(a,a)| a belongs to A}

s(R) = symmetric closure of R

= R union R^{-1}

R^{-1} = {(y, x) | (x, y) is in R}

t(R) = R union R^2 union R^3 union …

$R^{k =}$ is the relation you get when you compose R with itself k-1 times.

A = {1,2,3}, B = {a,b,c}, C = {w,x,y,z}

R = {(1,a), (2, a), (3, b), (3, c)}

S = {(a, w), (a, y), (c, x)}

S ° R = { (1, w), (1, y), (2, w), (2, y), (3, x) }

A = {1,2,3,4}

R = {(1,2), (1,3), (2, 4), (3, 1), (4, 2), (4, 3)}

R ° R = {(1,4), (1, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 1), (4, 4) }

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S = {}
For (x, y) in R
    For (w, z) in R
        If (y == w)
            S = S union {(x, z)}
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A = {1,2,3,4,5,6}

(a) How many relations are symmetric and reflexive
Must have (1,1) … (6, 6)
30 ordered pairs left → (1,2) and (2,1)
(2,5) and (5,2) (15 pairs of ordered pairs)
2 choices for each pair of pairs, so there are 2^{15} possible relations.

(b) How many symmetric relations contain (1,1), (3, 3), (2, 4) and (5, 1)?

Freedom choice with (2,2),(4,4),(5,5),(6,6) 2^4

(2, 4) and (4, 2) in set no choice

(5, 1) and (1, 5) in the set no choices

13 other pairs of pairs, like (1,2) and (2,1). For each of these 13 pairs of pairs, I can either put them in the relation or not → 2^{13}

Final answer = $2^4 \times 2^{13} = 2^{17}$

(c) How many anti-symmetric relations contain (2, 2), (3, 5) and (4, 6)?

(1,1), (3,3),(4,4),(5,5),(6,6) I have two choices 2^5

(3,5) is IN, (5,3) is OUT

(4, 6) is IN, (6, 4) is OUT

13 other pairs of pairs like (1, 2) and (2,1) for these two items, we can do the following : {}, {(1,2)}, {(2,1)} → 3^{13}

Final answer = $2^5 3^{13}$

Function stuff

Only one polynomial has roots 2, 3, 4 and is of degree 3 with a leading coefficient of 1, that polynomial is

$F(x) = (x – 2)(x – 3)(x – 4) = x^3 + ax^2 + bx + c$

What is the value of a – b + c?

F(1) = (1-2)(1-3)(1-4) = 1 + a + b + c → a+b+c = -7

F(-1) = (-1-2)(-1-3)(-1-4) = -60 = -1 + a – b + c → a – b + c = -59

Note f(0) reveals c. (constant coefficient)

How to find the inverse of a function?

Flip x and y and solve for y…

F(x) = 3x + 7

X = 3y + 7

(x – 7) = 3y

Y = (x – 7)/3

$f^{-1}(x) = (x – 7)/3$

Function maps an input from a domain to an output that's in a co-domain. A function is a specific type or relation (A is the domain, B is the co-domain)

$F(x) = x^2 + 4$… domain is Reals, co-domain is Reals, Range [4, inf)

In order for a function to be invertible, it has to be one-to-one, meaning that each output to the function maps to a different input…

If f(2) = 7 and f(3) = 7, what is $f^{-1}(7)$ = ????

In order for an inverse to exist, a function must be one-to-one (injective)

Injective definition: For all x and y in the domain, if f(x) = f(y), then x = y.

F(x) = (x – 2)/x, domain is all real x except x = 0, co-domain is all real x except x = 1.

(a – 2)/a = (b – 2)/b

1 – 2/a = 1 – 2/b

2/a = 2/b

2a = 2b

a = b…done

$f(x) = (x – 2)^2 + 4$, domain = all reals >= 2, range is all reals >= 4

$x = (y - 2)^2 + 4$

$(x - 4) = (y - 2)^2$

$\text{Sqrt}(x - 4) = y - 2$

$Y = 2 + \text{sqrt}(x - 4)$, because domain is >= 2. If domain had been <= 2, then this would be a minus sign.

A = {1, 2, 3, 4}

R = {(1,1), (2,2), (3, 3), (4,4)} is symmetric, transitive, anti-symmetric, reflexive

R = {(1,1)}, symmetric, anti-symmetric, transitive

R {(1,2), (1, 3), (1,4), (2, 3), (2, 4), (3, 4)} // aka less than

Irreflexive, anti-symmetric, transitive

R = {} // irreflexive, symmetric, anti-symm, transitive