Lec 4 Question 8

x = 1 – 1/5 – 2/5 – 1/10 = 3/10

p(6 in 2 rolls) = p(2)*p(4) + p(3)*p(3) + p(4)*p(2)

= 2*p(2)*p(4) + p(3)*p(3)

= 2*(4/10)*(3/10) + (1/10)*(1/10)

= 25/100 = ¼

P(3&3 | sum=6) = p(3&3 and 6 sum)/p(sum 6)

= (1/100)/(25/100) = 1/25

Question 9

$$k\sum_{x=0}^{\infty}(\frac{2}{3})^{x} = 1$$

$$\frac{k}{1-\frac{2}{3}} = 1$$

$$k = \frac{1}{3}$$

Question 10

P(Jack win first) = 4/6 = 2/3

P(Jill win first) = p(Jack push)*p(Jill wins) = (1/3)*(2/3) = 2/9

P(Jack wins) = 2/3 + (1/3)*(1/3)*P(Jack wins)

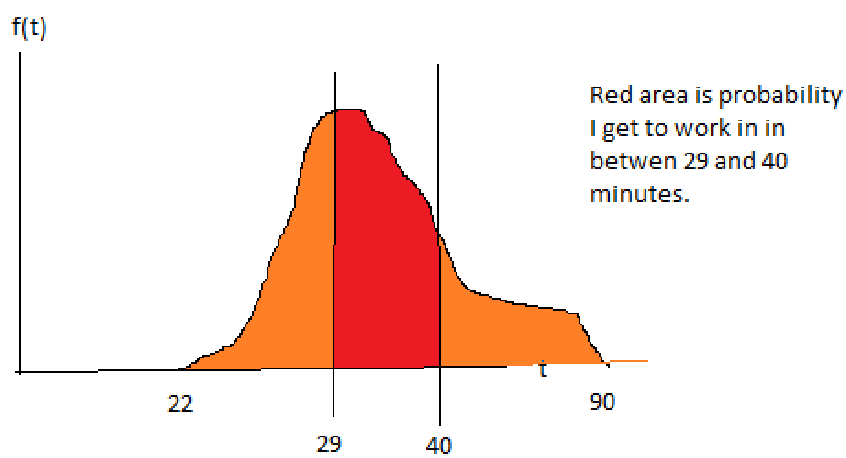
Either jack wins on the first turn, or both roll 5 or 6, returning the game to the beginning

P(Jack wins)*(1-1/9) = 2/3

P(Jack wins) = (2/3)*(9/8) = ¾

Continuous Probability Distribution

Instead of a variable taking discrete values each with some positive probability, we have a function f(x) which describes the probability such that the area under curve equals 1. The area of x being in between a and b is defined as $p(a \leq x \leq b) = \int_a^b f(x)dx$



$$E(x) = \int_{-\infty}^{\infty} xf(x)dx$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$Var(X) = \int_{-\infty}^{\infty} \left(x - E(X)\right)^2 f(x)dx$$

MODE = max value of f(x)

MEDIAN m = value of m such that $\int_{-\infty}^{m} f(x)dx = \frac{1}{2}$.

Question 11

(a) E(X) = $\frac{1}{12}\int_0^2 x(8x - x^3)dx = \frac{1}{12}\left(\frac{8}{3}x^3 - \frac{x^5}{5}\right)|_0^2 = \frac{1}{12}\left(\frac{64}{3} - \frac{32}{5}\right) = \frac{56}{45}$

(b) $\frac{1}{12}\int_0^m (8x - x^3)dx = .5$

$$\frac{1}{12}(4m^2 - \frac{m^4}{4}) = \frac{1}{2}$$

$$4m^2 - \frac{m^4}{4} = 6$$

$$16m^2 - m^4 = 24$$

$$m^4 - 16m^2 + 24 = 0$$

(c) $8 - 3x^2 = 0$ → $x = \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{6}}{3}$

Question 12 – skip for now, will write on paper at end

Lecture 5 Question 6

Prob of k heads is $\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$

$$\sum_{k\in Even}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$$

$$(\frac{2}{3}+\frac{1}{3})^{50}=\sum_{k=0}^{50}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$$

$$(\frac{2}{3}+\frac{1}{3})^{50}=\sum_{x\in Even}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}+\sum_{x\in Odd}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$$

$$(\frac{2}{3}-\frac{1}{3})^{50}=\sum_{k=0}^{50}\binom{50}{k}(\frac{2}{3})^k(-\frac{1}{3})^{50-k}$$

$$(\frac{2}{3}-\frac{1}{3})^{50}=\sum_{x\in Even}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}-\sum_{x\in Odd}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$$

Add these two equations to get:

$$1+(\frac{1}{3})^{50}=2\sum_{x\in Even}\binom{50}{k}(\frac{2}{3})^k(\frac{1}{3})^{50-k}$$

$$\sum_{x \in Even} \binom{50}{k} (\frac{2}{3})^k (\frac{1}{3})^{50-k} = \frac{1 + (\frac{1}{3})^{50}}{2}$$

Question 9

Sample Space # of divisors of $10^{99} = 2^{99}5^{99}$ → # div = 100 x 100 = 10000

How many of these are multiples of 10^{88} → 2^a5^b

88 <= a <= 99 (12 values)

88 <= b <= 99 (12 values)

Prob = $\frac{12 \times 12}{100 \times 100} = \frac{9}{625}$

Question 10

P(H = 1) = $\binom{5}{1} p(1-p)^4 = \binom{5}{2} p^2(1-p)^3 = P(H = 2)$

$$5p(1-p)^4 = 10p^2(1-p)^3$$

$$5(1-p) = 10p$$

$$5 - 5p = 10p$$

$$15p = 5 \rightarrow p = \frac{1}{3}$$

P(H=3) = $\binom{5}{3}(\frac{1}{3})^3(\frac{2}{3})^2 = \frac{40}{243}$

Question 11

Sample Space = 2^{10}

One way split work into 0H, 1H, 2H, 3H, 4H, 5H → combinations with repetition…(hard way)

F(n) = # of sequences of n coin tosses without consecutive heads

F(0) = 1

F(1) = 2

F(2) = 3

Consider calculating f(n) from previous values…

All sequences either end in T or H.

[Sequence length n-1]T

[Sequence of length n-2]TH

F(n) = f(n-1) + f(n-2)

F(3) = 5, f(4) = 8, f(5) = 13, f(6) = 21, f(7) = 34, f(8) = 55, f(9) = 89, f(10) = 144

Answer = 144/1024 = 9/64