COT 3100 4/14/23

New Topic: Probability

* Let's Make a Deal

How Low Can You Go?

[A] [B] [C]

Distinct randomly chosen ints range 1-10

A 5 C

Contestant Picks Box Value revealed

Contestant choose final door.

Goal: Pick smallest of the 3 numbers.

8-B	Switch	L
9-C	Switch	W
2-B	Stay	L
3-B	Stay	L
5-A	Switch	L
5-C	Switch	W
8-B	Switch	W
* 4-B	Switch	W
2-C	Stay	W
8-A	Switch	W

7-B	Switch	W
8-C	Switch	L
7-B	Switch	L
*6-A	Stay	W
5-C	Switch	L
3-A	Stay	W
7-C	Switch	W
8-B	Switch	L

What's the prob we'd get no duplicates in 18 times playing if I didn't screen them out?

Intuitively Stay low, Switch high

Optimal: value $\leq k \implies$ Stay

value $> k \implies$ Switch

Prob that given we've picked k Stay, what is the chance of winning

" " " " " " SWITCH, what's chance of winning?

$\boxed{1\ 2 \ldots}\ k\ \boxed{k+1 \ldots 10}$ Select 2 from these $\binom{9}{2}$ ways to do this.

$$p(\text{Win} \mid \text{STAY}) = \frac{\binom{10-k}{2}}{\binom{9}{2}} = \boxed{\frac{(10-k)(9-k)}{72}}$$

$$p(\text{Win} \mid \text{SWITCH}) = \frac{1 - p(W \mid \text{STAY})}{2}$$

$$= \frac{1}{2} - \frac{(10-k)(9-k)}{144}$$

$$\frac{(10-k)(9-k)}{72} \geq \frac{1}{2} - \frac{(10-k)(9-k)}{144}$$

$$2(10-k)(9-k) \geq 72 - (10-k)(9-k)$$

$$3(10-k)(9-k) \geq 72$$

$$(10-k)(9-k) \geq 24$$

$$90 - 19k + k^2 \geq 24$$

$$k^2 - 19k + 66 \geq 0$$

$$k = \frac{19 \pm \sqrt{19^2 - 4(66)}}{2} = \frac{19 \pm \sqrt{97}}{2}$$

$$\frac{19 - \sqrt{97}}{2} \sim 4.6$$

$$\boxed{k \leq 4}$$

Most simple type of problem:

Rolling fair six-sided die, win roll 1 or 6

Sample Space = 6 sides (equally likely)
POSSIBLE OUTCOMES

Successes = 2

$$\text{prob} = \frac{\text{SUCCESS}}{\text{SAMPLE SPACE}} = \frac{2}{6} = \frac{1}{3}$$

All probs ≥ 0 and probs ≤ 1.

10 blue socks, 6 red, 8 green
sock at random, what's prob it's green

$$\frac{8}{6+8+10} = \frac{8}{24} = \frac{1}{3}$$

Roll 2 dices sum showing $[2, 12]$ not equally likely

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$p(2) = \frac{1}{36}$$

$$p(7) = \frac{6}{36}$$

$$p(k) = \frac{6 - |k-7|}{36}$$

prob rolling k $2 \leq k \leq 12$
sum of 2 dice

2 → 1
12

3 → 2
11

6,8 → 5
7 → 6

Lottery

$1, 2, \ldots, 52$ $\quad$ $7, 22, 23, 40, 47, 51$]

Choose 6

Match 3, 4, 5 or 6 $\Rightarrow$ WIN

What's the probability of matching k numbers

Sample Space $= \binom{52}{6}$

\# ways to correctly match exactly k #s

Choose k from 6 correct

Choose 6-k from 46 inc. values

incorrect

$$= \binom{6}{k} \times \binom{46}{6-k}$$

$$\text{prob} = \frac{\binom{6}{k}\binom{46}{6-k}}{\binom{52}{6}}$$

$p(A)$ = prob of event A occurring

$p(\bar{A}) = 1 - p(A)$, prob of A not occurring

$p(A \cap B)$ = prob of A and B occurring

$p(A \cup B)$ = prob of A or B occurring

2 events mutually exclusive: $p(A \cap B) = 0$.

$\hookrightarrow p(A \cup B) = p(A) + p(B)$

2 events independent, then $p(A \cap B) = p(A) \times p(B)$

$p(A \mid B) =$ "probability of event A occurring given that event B has occurred"

$$= \frac{p(A \cap B)}{p(B)}$$

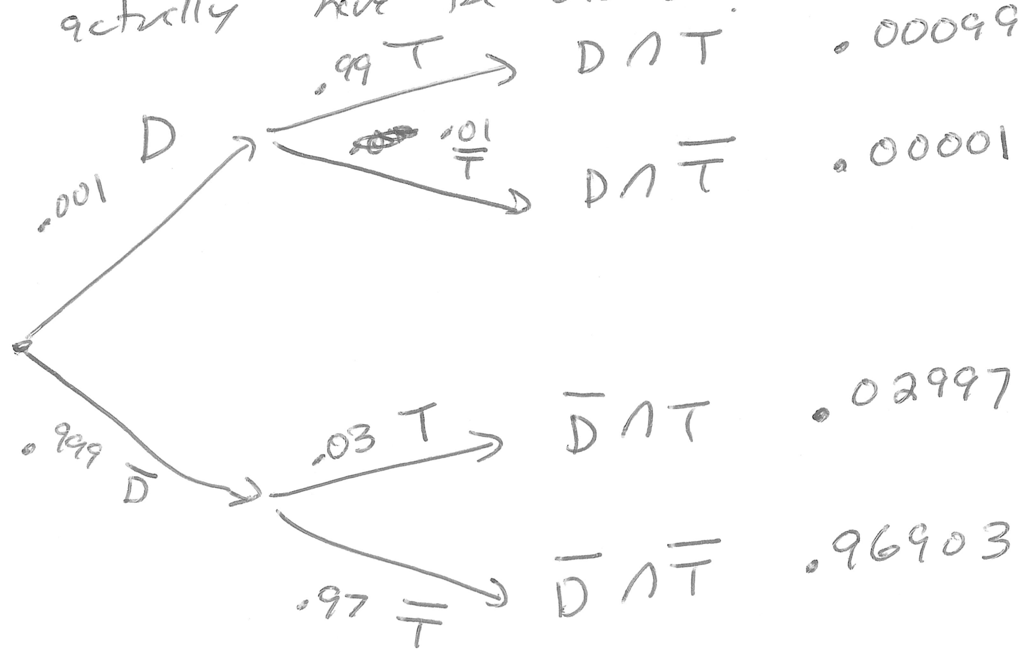
Independent $\rightarrow$ $p(A \mid B) = p(A)$.

Disease .1% of people $\quad p(D) = .001$

$p(T \mid D) = .99$

$p(\bar{T} \mid \bar{D}) = .97$ * $\boxed{\text{FALSE POSITIVE RATE 3\%}}$

$p(D \mid T) =$ "I tested positive, what is the probability I actually have the disease?"



$$p(D \mid T) = \frac{p(D \cap T)}{p(T)} = \frac{p(D \cap T)}{p(D \cap T) + p(\bar{D} \cap T)}$$

$$= \frac{.00099}{.00099 + .02997} = \frac{99}{3096}$$

$$\cong 3.2\%$$

Bayes Law