COT3100 2/7/2023

Announcements

1) Do NOT come to class 2/9/23
   - Zoom Video (2/8)
   - Post Webcourses by noon
   - Watch in between 2/8 and 2/10

2) Exams Returned

   M 8am - I have, set @ end to today's class

   Everyone else - Recitation!

Agenda

(1) Exam Feedback

(2) Num Theory
   - revamp typed notes

Exam 1 - Question by Question Avg

W	1 -	88.5%	Truth Table
↑	2 -	70.9%	Rules Inf.
Could use improvement	3 -	26.9%	∀∀ quantifier completing square
	4 -	40.0%	Inclusion/Exclusion
	5 -	37.3%	Set proof
	6 -	37.9%	Disproof set
	7 -	49.0%	D=RT
	8 -	45.4%	D=RT
	9 -	38.3%	log
	10 -	100%	:)

Q3

Algebra - completing square
partial fraction $\frac{x-1}{x} = 1 - \frac{1}{x}$

Q4

Writing out I/E principle good idea
circling what you know
figuring out ~~what you need~~
~~$|A \cup B|$~~ - $|A \cap B| = \boxed{|A| + |B| - |A \cup B|}$

Q5

Learn how to set up Proof by contradiction

Q6

Disproof specify example exactly
Make if true
then false

$$② \frac{15}{\frac{25}{12}} = \frac{\cancel{15}^{3} \times 12}{\cancel{25}_{5}} = \frac{36}{5} = \boxed{7.2}$$

Q7 + Q8

1) Minimize the # of variables you create!
2) USE FRACTIONS! fractions are your friend.
3) Pay attention units of measurement (minutes ⟹ hours)

Q9

challenging

step1 - change base → same

$$\log_9 3x^2 \neq 2\log 3x$$

$$\log_9 3x^2 = \log_9 3 + \log_9 x^2 = \frac{1}{2} + 2\log_9 x$$

$$\log_9 (3x)^2 = 2\log_9 3x$$

early as possible substitute $a = \log_3 x$, $b = \log_3 y$

Distribution

75	3
65-74	14
55-64	18
45-54	34
35-44	38
25-34	50
15-24	34
5-14	21

Number Theory

Gauss - "Math is the queen of the sciences. Number Theory is the queen of math."
↳ "Study of properties of integers"

"$a|b$" - "b is divisible by a"

$\exists c \in \mathbb{Z} \mid b = ac$

$7|14$, $101|0$, $13|13$, $24 \nmid 12$

① $\forall a \in \mathbb{Z}$, $1|a$

② $\forall a \in \mathbb{Z}$ except $a=0$ $a|0$

③ $\forall a,b,c \in \mathbb{Z}^+$ if $a|b \wedge b|c$, then $a|c$.

④ $\forall a,b \in \mathbb{Z}$, $a,b \neq 0$, if $a|b \wedge b|a$, then $a = \pm b$

⑤ if $a|y \wedge a|z \wedge x = y+z$, then $a|x$.

⑥ if $a|b \wedge a|c$ $\forall x,y \in \mathbb{Z}$ $a|(bx+cy)$.

Find all integer solutions (x,y) to the equation $5x + 10y = 132$.

Use rule ⑥ $\Rightarrow 5 \mid (5x+10y)$

but $5 \nmid 132$

There can be no integer solutions to this equation since it's impossible for the same number to be divisible by 5 and not divisible by 5.

Let x, y be integers such that $13 \mid (3x+4y)$.

Prove $13 \mid (7x+5y)$

*Creative step!

$$7x + 5y = 13x + 13y - 2(3x+4y)$$

$$= 13(x+y) - 2(13c), \quad \exists c \in \mathbb{Z} \mid 3x+4y = 13c$$

$$= 13(x+y-2c)$$

Since $x, y, c \in \mathbb{Z}$, it follows that $x+y-2c \in \mathbb{Z}$.

Thus, $13 \mid (7x+5y)$ as desired.

Def Division

Given two positive integers a and b there is a unique way to divide a by b to obtain a quotient (q) and remainder (r) with $0 \leq r < b$, integers q, r

$$a = bq + r$$

$$\begin{array}{r} 9 \text{ R5} \\ 13 \overline{)122} \\ -117 \\ \hline 5 \end{array}$$

$$\boxed{\begin{array}{c} a = 122,\ b = 13 \\ q = 9,\ r = 5 \end{array}}$$

$$\boxed{\begin{array}{c} a = 7 \quad b = 103 \\ q = 0,\ r = 7 \end{array}}$$

There is at least 1 solution w/o restriction on r:

$\cancel{q}\ \cancel{b} = 0, \quad r = a$

$a = b(0) + a$ ✓

let (q, r) be the solution with the min value of $r \geq 0$.

Assume the opposite, $r \geq b$

$$a = bq + r, \qquad r \geq b$$

$$= bq + b + r - b$$

$$= b(q+1) + (r - b)$$

$$q' = (q+1),\ r' = r - b \geq b - b = 0$$

(q', r') is another solution w/ $r' \geq 0$
$r' < r$.

Contradicts assumption that r was minimal.

Prove uniqueness

$a = bq + r \qquad 0 \le r < b$

Can we get a second solution with r' w/ $0 \le r' < b$ and $r' \neq r$

Assume the opposite that a second solution exists

$a = bq + r = bq' + r'$

$bq - bq' = r' - r$

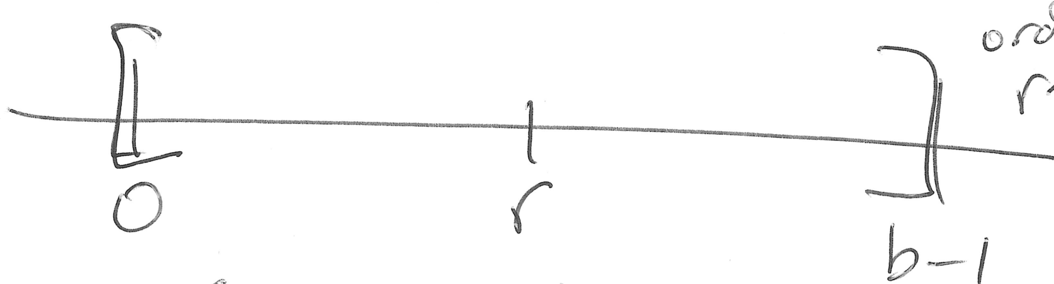
$b(q - q') = r' - r$

~~either~~ $\Rightarrow b \mid (r' - r)$

either $r' - r = 0$ not allowed

or $|r' - r| \ge b$

r has a restraining order on r'
r' can't get within b feet of r.



Can r' be inside the brackets?

Base Conversion

$\underline{325}_{\underline{6}} = 3\times6^2 + 2\times6^1 + 5\times6^0 = 3\times36+12+5 = 125_{10}$

base 10 $\quad 2023 = 2\times10^3 + 0\times10^2 + 2\times10^1 + 3\times10^0$

to convert base b to base 10

Convert $\underline{125}_{10}$ to base 6

6	125	
6	20	R 5
6	3	R 2
	0	R 3

$125 = \underbrace{d_2\times6^2 + d_1\times6^1} + d_0\times6^0$

$125 = 6\underbrace{(d_2\times6^1 + d_1\times6^0)}_{q} + \underbrace{d_0}_{r}$

need to recursively convert q to base b