COT3100 1/31/23

① I/E

② More Examples of Set Proofs

③ Exam Review (for Thursday)

1st problem: typed on next page

Example #2

$|A \cap C| = 7$ Determine $|B|$.

$|A \cap B \cap C| = 3$

$|A \cup B| = 20$

$|A \cap B| = 8$

$|B \cup C| = 22$

$|A \cup B \cup C| = 23$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$20 = |A| + |B| - 8$$

$$\boxed{|A| + |B| = 28}$$

$$* \boxed{|B \cap C| = |B| + |C| - |B \cup C|}$$

$$|A \cup \underline{B \cup C}| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - \underline{\underline{|B \cap C|}} + |A \cap B \cap C|$$

$$23 = \underline{28} + |C| - \underline{8} - \underline{7} - [|B| + |C| - |B \cup C|] + \underline{3}$$

$$23 = 16 + \cancel{|C|} - |B| - \cancel{|C|} + |B \cup C|$$

$$23 = 16 - |B| + 22$$

$$|B| = 16 + 22 - 23 = 15$$

$$* \quad |B \cup C| = |B| + |C| - |B \cap C| \rightarrow |B \cap C| = |B| + |C| - |B \cup C|$$

$$- |B \cup C| \qquad - |B \cup C|$$

$$+ |B \cap C| \qquad + |B \cap C|$$

Let's use the Inclusion-Exclusion Principle to deal with a counting problem involving sets:

David owns a box full of blocks which come in two colors(red,blue), two sizes(small, large), and two weights(light, heavy). He owns each possible combination of block. The total number of blocks that are red or small or light is 25. Of these, exactly 13 are small, 5 are both small and red, and 3 are red, small and light. Also, exactly 20 blocks are either red or light. But only 7 blocks are red and light. There is a total of 14 red blocks. Finally, of all the blocks 18 are not light. Find the following pieces of information:

1) Total number of blocks that are either red or small $|A \cup B| = 22$
2) Total number of light blocks $|C| = 13$
3) Total number of blocks that are small and light $|B \cap C| = 6$

Let A be the set of red blocks, B be the set of small blocks, and C be the set of light blocks.

Using the given information, we have:

$|A \cup B \cup C| = 25$
$|B| = 13$
$|A \cap B| = 5$
$|A \cap B \cap C| = 3$
$|A \cup C| = 20$
$|A \cap C| = 7$
$|A| = 14$

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 14 + 13 - 5$$
$$= 22$$

$$|A \cup C| = |A| + |C| - |A \cap C|$$
$$20 = 14 + |C| - 7$$
$$20 = 7 + |C|$$
$$|C| = 13$$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$
$$25 = \underline{14 + 13 + 13} - \underline{5 - 7} - |B \cap C| + \underline{3}$$
$$25 = 31 - |B \cap C|$$
$$|B \cap C| = 31 - 25 = 6$$

1) $A \cup B$, 2) $A \cup C$, 3) $A \cap C$

Prove: if $A \subseteq B$, then $A \cap C \subseteq A \cap B$.

Let's use direct proof.

Let $x \in A \cap C$, arbitrarily chosen. Our goal is to show $x \in A \cap B$.

By def intersection $x \in A \wedge x \in C$.

Since $A \subseteq B \wedge x \in A$, by def subset $x \in B$.

Since $x \in A \wedge x \in B$ by def intersection $x \in A \cap B$.

Prove: if $B = A \cap B$, then $B \subseteq A$.

Let's use direct proof.

Let $x \in B$, arbitrarily chosen. Our goal is to show $x \in A$.

Since $x \in B$ and $B = A \cap B$, $x \in A \cap B$, since both sets are equal.

By definition of intersection $\boxed{x \in A} \wedge x \in B$.

By conjunctive simplification $x \in A$, as desired.

Disprove: if $(A \cup B) \subseteq (C \cup D)$, then $A \subseteq C \vee A \subseteq D \vee B \subseteq C \vee B \subseteq D$.

$A = \{1, 2\}$

$B = \{3, 4\}$

$C = \{1, 3\}$

$D = \{2, 4\}$

$A \cup B = \{1, 2, 3, 4\}$

$C \cup D = \{1, 2, 3, 4\}$

but $A \not\subseteq C$, $A \not\subseteq D$, $B \not\subseteq C$, $B \not\subseteq D$

$2 \in A \wedge 2 \notin C$	$1 \in A \wedge 1 \notin D$	$4 \in B \wedge 4 \notin C$	$3 \in B$ $3 \notin D$

Prove: if $A \subseteq B$, then $(C-B) \cap A = \emptyset$

Let's prove via contradiction.
Assume the opposite, that $(C-B) \cap A \neq \emptyset$.
Then $x \in (C-B) \cap A$, for some element x.
By definition set intersection, $x \in C-B \wedge x \in A$
By definition set difference, $x \in C \wedge x \notin B$.
If $x \in A \wedge x \notin B$, the by def subset $A \not\subseteq B$,
but this directly contradicts our given information
that $A \subseteq B$. It follows that our initial assumption
was incorrect, thus, $(C-B) \cap A = \emptyset$, as desired.

If $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

If we prove $A \subseteq B$ due to the symmetry of the problem,
it follows that $B \subseteq A$ as well, thus we can conclude $A = B$.

Goal: Prove $A \subseteq B$
Let $x \in A$, arbitrarily chosen. We want to show $x \in B$.
2 cases $x \in C \vee x \notin C$

Case 1: $x \in C$

$x \in A \wedge x \in C$
thus $x \in A \cap C$ by def intersection
Since $A \cap C = B \cap C$, $x \in B \cap C$
by def of set equality
By def intersection $\boxed{x \in B} \wedge x \in C$
We've proven $x \in B$ as desired.

Case 2: $x \notin C$

$x \in A \wedge x \notin C$
By def union $x \in A \cup C$.
Since $A \cup C = B \cup C$, $x \in B \cup C$
By def union, $x \in B \vee x \in C$.
But since $x \notin C$, it must be the case that $\boxed{x \in B}$ as desired.