COT 3100 1/24/23

Fun Prob of Day

	2	2	1	5	3	5
3	1	2	3	4	5	6
4	7	8	9	10	11	12
3	13	14	15	16	17	18
2	19	20	21	22	23	24
4	25	26	27	28	29	30
2	31	32	33	34	35	36

Some #s are circled. Row labels = # of values on row circled, Col labels = # of values on col circled.

WHAT IS SUM OF CIRCLED VALUES?

Principle = "Sometimes, you don't need to find the value of each variable in a problem to answer the posed question"

	2	2	1	5	3	5
3	1+0	1+1	1+2	1+3	1+4	1+5
4	7+0	7+1	7+2	7+3	7+4	7+5
3	...					
2						
4						
2	31+0	31+1	31+2	31+3	31+4	31+5

3×1
4×7
3×13
2×19
4×25
$+\ 2 \times 31$

$2 \times 0 +$
$2 \times 1 +$
$1 \times 2 +$
$5 \times 3 +$
$3 \times 4 +$
$+\ 5 \times 5$

(1,0), (1,1), (1,2), (1,3) (1,4) (1,5)
(7,0), (7,1), (7,2), (7,3), (7,4), (7,5)

Sets

Logic (1st 2 wk)

Sets (next 2 wks)

↳ collection of objects w/o duplicates.

$A = \{1, 3, 4, 6, 8\}$

element separate w/commas

$1 \in A$

"element of"

$2 \notin A$

multisets keep track of duplicates.

$E = \{2x \mid x \in \mathbb{Z}\}$

all elements of the form $2x$

"such that"

x is an element of the integers.

$= \{\ldots, -4, -2, 0, 2, 4, \ldots\}$

even integers

$O = \{2x+1 \mid x \in \mathbb{Z}\}$

Announcements

1) Study Groups sign up
2) Hw 1 turned in
   Solutions Posted

$\mathbb{Z}$ = int

$\mathbb{Z}^+$ = pos int

$\mathbb{Z}^-$ = neg int

$\mathbb{Q}$ = rational

$\mathbb{R}$ = real

$\mathbb{C}$ = complex

$\mathbb{N}$ = non neg ints

$A \subseteq B$

A is a subset of B

$\forall x \in A \rightarrow x \in B$

$A = \{1, 3, 5, 6\}$
$B = \{1, 2, 3, 5, 6, 8, 12\}$
$C = \{1, 3, 5, 6\}$
$D = \{1, 2, 3, 4, 5, 8, 10, 12, 15, 18\}$

$A = C$
$A \subseteq A$
$A \subseteq B$ ✓
$A \subseteq C$ ✓
$A \nsubseteq D \begin{pmatrix} 6 \in A \wedge \\ 6 \notin D \end{pmatrix}$

In order for $A = B$, $A \subseteq B \wedge B \subseteq A$

$\forall x \in A \rightarrow x \in B \wedge$
$\forall x \notin A \rightarrow x \notin B$

logically equivalent to
$\forall x \in B \rightarrow x \in A$

if we want to prove $A \neq B$ we must show either
(a) $\exists x \mid x \in A \wedge x \notin B$
or (b) $\exists x \mid x \in B \wedge x \notin A$

$A \subset B \longleftrightarrow A \subseteq B \wedge A \neq B.$

A is a proper subset of B

from above $A \not\subset A$
$A \subset B$
$A \not\subset C$
$A \not\subset D$

$|A|$ = # elements in A

cardinality of set A

$|A| = 4$ $|D| = 10$
$|B| = 7$
$|C| = 4$

Set Operators

$A \cup B = \{x \mid x \in A \vee x \in B\}$

"union"

$A = \{1,3,4,7\}$

$B = \{2,3,5,7,9\}$

$A \cup B = \{1,2,3,4,5,7,9\}$

$A \cap B = \{x \mid x \in A \wedge x \in B\}$

$A \cap B = \{3,7\}$

$\overline{A} = \{x \mid x \notin A\}$

complement of A

all element x not in A.

$\mathcal{U}$ = universe from which all elements are taken

$A - B = \{x \mid x \in A \wedge x \notin B\} = \underbrace{A \cap \overline{B}}_{\text{def set difference}}$

set difference

$A - B = \{1,4\}$ → Intuitively, take items in A but remove anything that's in B.

$B - A = \{2,5,9\}$

$A \oplus B = (A - B) \cup (B - A)$

symmetric difference

↳ Intuitively, it's every element that belongs exactly to one of the 2 sets A, B.

(basically "like" an xor)

Types of Problems

1) Two Sets ARE EQUIVALENT
   - Set Table Membership
   - Set Laws

Going to appear just like a truth table & using logic laws, but they are different in what they represent.

$$A = (A - B) \cup (A \cap B)$$

A	B	$A - B$	$A \cap B$	$(A-B) \cup (A \cap B)$
0	0	0	0	0
0	1	0	0	0
1	0	1	0	1
1	1	0	1	1

"let x be an arbitrarily chosen element such that $x \notin A \wedge x \notin B$, ..."

Proof by Cases

p, q, r 8 rows

p	q	r	$p \vee r$
0	0	0	0
0	0	1	
0	1	0	0

Set Laws

Look @ Exam 1 Ref Sheet

Prove $(A-B) \cup (A \cap B) = A$

Same

$\emptyset = \{\}$

$\{\emptyset\} \neq \emptyset$

$|\emptyset| = 0$

$|\{\}| = 0$

$|\underbrace{\{\emptyset\}}_{1 \text{ item}}| = 1$

$(A-B) \cup (A \cap B)$	Given
$(A \cap \overline{B}) \cup (A \cap B)$	Def Set Diff
$A \cap (\overline{B} \cup B)$	Distributive Law
$A \cap \mathcal{U}$	Inverse Law
A	Identity Law

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if $A \cap B = \emptyset$, then $B \subseteq \overline{A}$.

1. Direct Proof
2. Proof of Contrapositive
3. Proof by Contradiction

Direct Proof

We must show that $B \subseteq \overline{A}$. By def subset we want to prove that for an arbitrarily chosen element $x \in B$, that ~~$x \notin$~~ $x \in \overline{A}$.

(1) Let $x \in B$, arbitrarily chosen

(2) Since $A \cap B = \emptyset$ (given) and $x \in B \Rightarrow x \notin A$ because if $x \in A$, then $A \cap B \neq \emptyset$.

(3) By def set complement if $x \notin A \Rightarrow x \in \overline{A}$. ✓

Pf of Contrapositive

$p \to q$ Contrapositive $\overline{q} \to \overline{p}$
$\longleftrightarrow$

If $B \not\subseteq \overline{A}$, then $A \cap B \neq \emptyset$.

Use direct proof.

1) $B \not\subseteq \overline{A}$, Given
2) By definiton of subset
   $\exists x \mid x \in B \land x \notin \overline{A}$.
3) By def of set complement $x \in \overline{\overline{A}}$.
4) By double negation, $x \in A$.
5) By def of set intersection $x \in A \cap B$.
6) Since $x \in A \cap B$, $A \cap B \neq \emptyset$, as desired.

Pf by Contradiction

If $A \cap B = \emptyset$, then $B \subseteq \overline{A}$

Assume the opposite, that $B \not\subseteq \overline{A}$.
Therefore, $\exists x \mid x \in B \land x \notin \overline{A}$.
By def set comp, $x \in \overline{\overline{A}}$.
By ~~def~~ double negation, $x \in A$.
Thus $x \in A \cap B$ by def set intersection but this contradicts our given information that $A \cap B = \emptyset$. This means our initial assumption must be incorrect. It follows that $B \subseteq \overline{A}$ as desired.