Announcements

(1) Study Groups - Required
≥ 10 Diag Quiz $\Rightarrow$ 4 (SG1)
< 10 or didn't take $\Rightarrow$ Sign Up Study Group

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(2) Hmk 1 Due Sunday
typed word or LaTex
Start early
If you handwrite max grade = 25/50

Finish laws of logic
- truth table w/ boolean op
- laws of logic
- simplified logical expressions
- how to prove laws of logic via truth table

TODAY

1) Rules of Inference + how to use to build an argument
2) $\forall, \exists$ quantifiers
   couple practice Qs
3) Nested quantifiers

Modus Ponens

Assumptions 1. p "it's raining"

2. $p \to q$ "if it's raining, then ground is wet"

$\therefore q$ The ground is wet.

$$[p \wedge (p \to q)] \to q$$

Prove Modus Ponens in 2 ways:

(a) truth table

(b) Laws of Logic

p	q	$p \to q$	$p \wedge (p \to q)$	q	$[p \wedge (p \to q)] \to q$
F	F	T	F	F	T
F	T	T	F	T	T
T	F	F	F	F	T
T	T	T	T	T	T

"tautology"

$$[p \wedge (p \to q)] \to q \iff T$$

$[p \wedge (p \to q)] \to q$ Given

$[p \wedge (\overline{p} \vee q)] \to q$ Def Implication

$\overline{(p \wedge (\overline{p} \vee q))} \vee q$ Def Implication

$\overline{p} \vee (\overline{\overline{p} \vee q}) \vee q$ De Morgan's

$\overline{p} \vee (\overline{\overline{p}} \wedge \overline{q}) \vee q$ De Morgan's

$\boxed{\overline{p} \vee (p \wedge \overline{q})} \vee q$ Double Negation

$((\overline{p} \vee p) \wedge (\overline{p} \vee \overline{q})) \vee q$ Distributive

$(T \wedge (\overline{p} \vee \overline{q})) \vee q$ Inverse Law

$(\overline{p} \vee \overline{q}) \vee q$ Identity Law

$\overline{p} \vee (\overline{q} \vee q)$ Associative Law

$\overline{p} \vee T$ Inverse Law

T Domination Law

$x^2 - 2x - 3 = 0$ and $\boxed{x < 0}$ q

what is x?

$x^2 - 2x - 3 = 0 \longrightarrow p$

$(x-3)(x+1) = 0$ $\quad$ $x-3=0$ $\quad$ $x+1=0$

$\cancel{x=3}$ or $x = -1$ $\quad$ r $\quad$ s

$\boxed{x=3}$ $\quad$ $\boxed{x=-1}$

1. p Given
2. q Given
3. $p \to (r \vee s)$ Given (math step)
4. $q \to \overline{r}$ Given (basic logic)

$\therefore s$

1. p Given

2. $p \to (r \vee s)$ Given

→ 3. $r \vee s$ Modus Ponens w/1,2

4. q Given

5. $q \to \overline{r}$ Given

→ 6. $\overline{r}$ Modus Ponens w/4,5

7. s Disjunctive Syllogism w/3,6

Given:

1. $p \vee q$
2. $p \to s$
3. $q \to r$
4. $\overline{r}$
5. $s \to (t \wedge u)$

$\therefore t$

1. $p \to s$ Given
2. $q \to r$ Given
3. $p \vee q$ Given
4. $s \vee r$ Constructive Dilemma w/1,2,3
5. $\overline{r}$ Given
6. s Disjunctive Syllogism 4,5
7. $s \to (t \wedge u)$ Given
8. $t \wedge u$ Modus Pones w/6,7
9. t Conjunctive Simplification 8

Quantifiers

Universe

$\forall x \in \mathbb{Z}^+ [x > 0] \rightarrow p(x)$

$\boxed{\begin{array}{l} p(3) \text{ true} \\ p(-2) \text{ false} \end{array}}$

"for all" x that is an element of the positive integers $x > 0$.

for all statement is only true if when you ~~stas~~ plug in each possible value from the given universe, the open statement is true

Open Start Table

x	$p(x)$
0	F
1	T
-1	F
2	T
-2	F

Universe $\mathbb{Z}^+$

x	$p(x)$
1	T
2	T
3	T
4	T
5	T
$\vdots$	$\vdots$

only true if all the table is true.

$\exists x \in \mathbb{Z}^+ [x^2 = x] \quad q(x)$

"There exists" a value of x, where x is a positive integer with $x^2 = x$.

x	$q(x)$
1	T
2	F
3	F
4	F
$\vdots$	$\vdots$

true as long as there is at least 1 T.

Prove there exists - find one example!

*Disprove a for all - find one counter-example.

Prove a for all - Take an arbitrarily chosen element from the universe and prove the statement for that arbitrarily chosen element (Universal Generalization)

If I want to prove something about all prime numbers, I can't just prove it for 7 or 11, but I have to prove it for p, where p is an arbitrarily chosen prime number.

To disprove a there exists, you have to show that it can't happen at all which largely means proving the stmt false for an arbitrarily chosen element.

$$\overline{\exists x\, p(x)} \Longleftrightarrow \forall x\, \overline{p(x)}$$

$$\overline{\forall x\, p(x)} \Longleftrightarrow \exists x\, \overline{p(x)}$$

Prove or disprove

Universe of Real Numbers ($\mathbb{R}$)

$$\exists x \in \mathbb{R} \left[x^2+4x+3 > 2x^2+5x+6 \right]$$

$$-x^2-x-3>0$$

$$\frac{1}{4}-(x^2+x+\frac{1}{4})-3>0$$

$$-(x+\frac{1}{2})^2-\frac{11}{4}>0 \;?$$

This stmt is false. We must prove that no real value of x makes this true

alternatively, we must prove that for all x

$$-x^2-x-3<0$$

$$\begin{aligned}
-x^2-x-3 &= -(x^2+x\quad)-3 \\
&= -(x^2+x+\tfrac{1}{4})-3+\tfrac{1}{4} \\
&= -(x+\tfrac{1}{2})^2-\tfrac{11}{4} \\
&\le 0-\tfrac{11}{4} \\
&= -\tfrac{11}{4} \\
&< 0.
\end{aligned}$$