Spring 2023 COT 3100H Quiz #1 Solutions (2/27/2023)

1) (5 pts) Jazira lives 15 miles from work. If she drives at an average speed or ***r*** miles an hour, she arrives to work on time. If she drives ***r* – 10** miles an hour instead, she'll arrive at work 24 minutes late. What is the value of ***r***, in miles per hour?

Let t be the amount of time in hours it would take for Jazira to make it to work on time. Note that 24 minutes is 2/5 of an hour. The given information can be summarized in these two equations:

$$15 = rt$$

$$15 = (r - 10)\left(t + \frac{2}{5}\right) = rt - 10t + \frac{2}{5}r - 4 = (15 - 4) - 10t + \frac{2}{5}r$$

Using the second equation (we already substituted rt = 15), we can now get the following:

$$\frac{2}{5}r - 4 - 10t = 0$$

Recall that $t = \frac{15}{r}$ and that neither is 0, so we can multiply through by r:

$$\frac{2}{5}r - 4 - \frac{150}{r} = 0$$

$$\frac{2}{5}r^2 - 4r - 150 = 0$$

$$2r^2 - 20r - 750 = 0$$

$$r^2 - 10r - 375 = 0$$

$$(r - 25)(r + 15) = 0$$

Since r > 0, **r = 25 miles per hour.**

Grading: 1 pts for creating two equations (or equivalent) based on information
1 pts to use two equations to come down to an equation with 1 variable
2 pts to factor/quadratic that equation correctly
1 pt to extract the correct answer.

2) (5 pts) Determine, to two decimal places, the value of $(log_{10}20)(log_{10}5)$, given that $log_{10}2 = .30$, rounded to two decimal places. (Note: if you don't given information, you are doing the problem wrong.)

$$(log_{10}20) \times (log_{10}5) = (log_{10}(10 \times 2)) \times \left((log_{10}(\frac{10}{2})\right)$$

$$= (log_{10}10 + log_{10}2)(log_{10}10 - log_{10}2)$$

$$= (1 + .3)(1 - .3) = 1.3 \times .7 = \mathbf{.91}$$

Grading: 2 pt rewrite logs in terms of 10 and 2, 2 pts plug in 1 and .3 appropriately, 1 pt multiply correct answer.

3) (5 pts) Let $a_1, a_2, a_3, \ldots, a_{500}$ be an arithmetic series with a sum of 100,000.

If $\sum_{i=1}^{250} a_{2i} = 75{,}000$, what is the common difference of the sequence?

Using the given information, it follows that $\sum_{i=1}^{250} a_{2i-1} = 100{,}000 - 75000 = 25000$. Namely, if the even indexed terms sum up to 75,000, the odd indexed terms must equal the rest (difference of whole sum from sum of even terms). Now, subtract this equation (sum of a_{2i-1}) from the given equation (sum of a_{2i}):

$$\sum_{i=1}^{250} a_{2i} - \sum_{i=1}^{250} a_{2i-1} = 75000 - 25000$$

$$\sum_{i=1}^{250} (a_{2i} - a_{2i-1}) = 50000$$

$$\sum_{i=1}^{250} d = 50000$$

$$250d = 50000$$

$$\boldsymbol{d = 200}$$

Grading: 5 pts for any correct solution (many ways to solve this).
3 pts for either noticing difference idea or setting up 2 equations in 2 variables.
2 pt for solving

4) (5 pts) The sum of four **distinct** prime numbers is 31. What is the **maximum possible value** of their product?

All primes but 2 are odd. The sum of four odd numbers is even, thus 2 must be in the list. Thus, we must search for all possibilities of 3 distinct odd primes that add to 29. Start with large primes…23 is too big because we don't have 2 other odd primes that add to 6. Here's the list:

19 + 7 + 3
17 + 7 + 5
13 + 11 + 5

From here, we can either brute force which product of 3 x 7 x 19, 5 x 7 x 17 or 5 x 11 x 13 is bigger, to find the final answer of 2 x 13 x 11 x 5 = **1430.**

Alternatively, it's fairly easy to prove that for positive integers x, y and z with x > y > z, that (x – z)(x + z) > (x – y)(x + y). (Proof left to the reader!) Using this rule, it's clear that 17 x 5 > 19 x 3, so the second solution listed creates a larger product than the first. Similarly, 13 x 11 > 17 x 7, thus the last solution creates a larger product than the second, and must be the correct solution. Thus, we can use this logic to only multiply out this set of primes to get the correct answer.

Grading: 2 pts for determining that 2 is in the set, 2 pts for listing out all other possibilities, 1 pt for picking the correct one and correctly multiplying out the primes.

5) (5 pts) It takes Jenny six hours to paint a house and Bob five hours to paint the same house. If Jenny, Bob and Malia all work together to paint that house, it would take them 2 hours to do so. How long would it take for Malia to paint this house on her own? **Please provide your answer in hours and minutes.** (ie 3 hours, 27 minutes)

Let M be the number of hours that it takes Malia to paint the house. Then we have:

$$\frac{2}{6} + \frac{2}{5} + \frac{2}{M} = 1$$

$$\frac{10 + 12}{30} + \frac{2}{M} = 1$$

$$\frac{11}{15} + \frac{2}{M} = 1$$

$$\frac{2}{M} = \frac{4}{15}$$

$$M = \frac{15}{2}$$

Converting to hours and minutes, we have **7 hours, 30 minutes.**
Grading: 2 pts set up equation, 2 pts solve for M, 1 pt convert to hours, minutes.