<u>COT 3100 Section 201 Exam #1 - Part 1 (Logic) - 25 pts (2/6/2023)</u>

Last Name: ___________________ , First Name: _____________________

1) (8 pts) Fill out the following truth table. Please place a T or F in each empty slot. Any ambiguous letter (according to the grader) will be marked incorrect.

p	q	r	$r \rightarrow p$	$\bar{q} \rightarrow p$	$(r \rightarrow p) \wedge (\bar{q} \rightarrow p)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (8 pts) Prove the following argument via the Rules of Inference. You may skip the Commutative Step in this problem. (It comes up once.)

$$p \rightarrow (s \vee t)$$
$$\bar{r} \wedge p$$
$$\bar{s} \vee r$$

$$\therefore t$$

Number	Step	Reason
1		
2		
3		
4		
5		
6		
7		
8		
9		

Note: You may not use all the rows in the table.

3) (9 pts) The n^th triangular number, denoted as T(n), is the sum of the first n positive integers. (Specifically, $T(n) = \frac{n(n+1)}{2}$.) If we write out the first few triangular numbers: 1, 3, 6, 10, 15, 21, 28, 36, … we notice a pattern: namely, the first two terms are odd, next two are even, next two are odd, etc. In particular, for any positive integer, n, the parity of $T(n)$ and $T(n+2)$ is always different. (Parity is whether a number is even or odd.) Thus, if $T(n)$ is even, then $T(n+2)$ is odd, and , if $T(n)$ is odd, then $T(n+2)$ is even. Prove this fact about Triangular Numbers, namely that for all positive integers n, the parity of $T(n)$ and $T(n+2)$ is always different. (Note: You can prove the two if then statements written above, or there's a nice way to prove the statement in a single proof by observing a more simple statement that is equivalent to the two if statements.)

<u>COT 3100 Section 201 Exam #1 - Part 2 (Sets) - 25 pts (2/6/2023)</u>

4) (10 pts) Let A = {3, 7} and let B = {2, 9, 12}. Consider the set $\wp(A \times B)$. (a) How many elements does $\wp(A \times B)$ have? (b) Imagine listing those elements (subsets) from smallest subset to largest subset (in size/cardinality). If two subsets are the same size, list the one that comes first lexicographically first. In determining lexicographical order of single elements (a, b) and (c, d), the element (a, b) comes before (c, d) if either a < c or if a = c and b < d. Sets should be listed in lexicographical ordering by the following definition: compare the first element in each set, then the next, until there is a discrepancy. Break the tie based on the discrepancy. For example, the set { (3, 9), (3, 12), (7, 2) } should be listed before { (3, 9), (3, 12), (7, 9) } because the first two elements of both sets are the same, but (7, 2) comes before (7, 9), the corresponding third elements. Given this ordering of the subsets in $\wp(A \times B)$, which subset shows up as the 58^{th} on the list?

a) ________ b) { }

5) (10 pts) Prove/Disprove: For sets A, B, C, D, if $A \subseteq B \cap C$ and $D \subseteq C \cap \bar{B}$, then $A \cap D = \emptyset$.

Please carefully indicate whether you are proving or disproving, followed by the appropriate justification for your answer. (The answer is worth 2 pts and the justification is worth 8 pts.)

6) (5 pts) How many integers in between 1 and 1000 are divisible by either 3 or 11?

COT 3100 Section 201 Exam #1 - Part 3 (D=rt, logs) - 25 pts (2/6/2023)

7) (8 pts) Ming bikes up a mountain and back down a mountain. Let r be her average speed biking on flat ground. When she bikes up the mountain, her average speed is 3 miles an hour less than when she's biking on flat ground. When she bikes down the mountain, her average speed is 3 miles an hour more than when she's biking on flat ground. Her round trip, which was 24 miles up the mountain and 24 miles back down the mountain took her 6 hours. What is her average rate of speed biking **on flat ground?**

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8) (6 pts) What is the value of the following product?

$$(log_3 x) \times (log_{47} 9) \times (log_x 47)$$

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9) (10 pts) Jim and Sang live 20 miles away from each other. At 2 pm, both start riding their bikes in the direction of the other person. Jim bikes at an average rate of 8 miles per hour while Sang bikes at an average rate of 7 miles per hour. When Jim starts biking, a little bee starts where Jim is, going towards Sang, at a rate of 39 miles per hour. As soon as the bee reaches Sang, it turns around and heads back towards Jim. The bee continues doing this until Jim and Sang meet. Assuming that the bee turns around instantaneously each time, constantly maintaining its same average speed in its direction of motion, how far did the bee fly from the time Jim and Sang started biking to the time they were at the same location?

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10) (1 pt) On this day (February 6) in 1952, Queen Elizabeth II ascended the throne of the UK and Northern Ireland. How many Queen Elizabeths were there before her?

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Scratch Page – Please clearly mark any work on this page you would like graded.