COT 3100H Final Exam - Part B (All Material except Relations, Functions) - 100 pts (4/26/2023)

Last Name: ___________________ , First Name: _____________________

1) (6 pts) Use the rules of inference to prove the following argument:

$$\bar{t} \rightarrow \bar{s}$$
$$p \rightarrow q$$
$$q \rightarrow (\bar{s} \vee r)$$
$$s \wedge p$$

$$\therefore r \wedge t$$

Please write each step one by one. More lines have been provided than necessary.

Number	Step	Reason
1		
2		
3		
4		
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6		
7		
8		
9		
10		
11		
12		

Note: You may not use all the rows in the table.

2) (8 pts) Define the set D = $\mathcal{R} - \{0\}$. Prove or disprove the following two statements, concerning this set:

(a) $\forall x \in D[\exists y \in D | xy > 0\}$

Circle whether the statement is true of false: TRUE FALSE

Prove your answer below:

(b) $\exists x \in D[\forall y \in D | xy > 0\}$

Circle whether the statement is true of false: TRUE FALSE

Prove your answer below:

3) (6 pts) Prove or disprove the following statement about finite sets, A, B, C and D:

$$\text{If } (A \cup B) \subseteq (C \cap D), \text{ then } (A \cap C) \subseteq (B \cap D)$$

4) (6 pts) Show the steps of any of the fast modular exponentiation algorithms to calculate the remainder with 13^{45} is divided by 29. (You may plug in multiplication problems into your calculator but must write out the result of each relevant multiplication and how you are going to combine them. The absolute value of all products calculated should not exceed 784.)

Remainder when 13^{45} is divided by 29: ____________

5) (8 pts) Prove or disprove the following statement about finite sets A, B and C:

If $A \cap C = A$, $B \cap C = B$ and $C - B \subseteq C - A$, then $A \subseteq B$.

6) (10 pts) Find the value of 61^{-1} mod 144 via the Extended Euclidean Algorithm.

7) (8 pts) Let T represent the arithmetic sequence a_1, a_2, a_3, …, with common difference d. Define T_n to be the sum of the first n terms of this sequence. It is given that $T_{2n} - 2T_n = 9n^2$. With proof, determine the value of d. (Your answer should be a number, not an expression with a variable.)

d = ________

8) (8 pts) If $x + \frac{1}{x} = 3$, what is the value of $x^4 + \frac{1}{x^4}$?

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9) (5 pts) How many permutations are there of the letters in the word COFFEECUP? (Please leave your answer in factorials.)

10) (10 pts) Jacinda wants to buy 20 flowers. The store she went to has roses, carnations, tulips, sunflowers and daisies. She has decided that she wants at least 3 roses and the store, unfortunately only has 4 tulips in stock. How many different combinations of flowers could she buy within these constraints? (Please leave your answer in combinations, factorials, etc.)

11) (10 pts) Let the probability of tossing heads on a biased coin be $\frac{3}{4}$. Let T_n be the probability of tossing an even number of heads after tossing the coin n times. Using induction on n, prove that $T_n = \frac{1+(-\frac{1}{2})^n}{2}$, for all non-negative values of n. (As you can see, I've discovered a much better way to word this problem!)

12) (8 pts) How many permutations of the whole alphabet of 26 letters do NOT have any consecutive vowels? (Note: the vowels are A, E, I, O, U.) (Please leave your answer in combinations, factorials, etc.)

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13) (5 pts) Jack and Jill play a game with a spinner with 3 sections of equal area marked with the labels 1, 2 and 3. The two alternate turns, with Jack going first. At the beginning of the game, a counter is set to 4. On a player's turn, they spin the spinner (each number is equally likely to come up), and that number is subtracted from the counter. A player wins if the value of their spin is equal to or greater than the counter. (For example, if Jack spins 1, Jill 2, and Jack gets 1, then Jack wins.) What is the probability Jack wins the game?

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14) (2 pts) What can be found on the inside of a Jelly Donut? ________________

Scratch Page - Please clearly mark any work on this page you would like graded.