COT 3100 Section 2 Exam #2 - Part 1 (Factorization, Algebra) - 15 pts (3/9/2023)

Last Name: ___________________ , First Name: _____________________

Circle Rec: M8am M4:30pm T10:30am W8am R10:30am R4:30pm F11:30am

1) (5 pts) What is the smallest positive integer N such that $720N = x^3$, where x is also a positive integer?

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2) (5 pts) How many integers in between 1000 and 9999, inclusive, have an odd number of positive integer divisors?

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3) (5 pts) Typically, Bob and Gary paint model houses together. (All model houses are identical, so if the same people are painting them, they take the same time to paint.) If Alice joins them, then they take six fewer hours to complete painting a model house. In fact, if Alice painted alone, she would finish painting a model house twice as fast as Bob and Gary working together! How long does it take Bob and Gary, working together, to paint a model house? (Hint: Just create a single variable, x, representing the number of hours it takes Bob and Gary, working together, to paint a model house.)

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COT 3100 Section 2 Exam #2 - Part 2 (Number Theory) - 25 pts (3/9/2023)

Last Name: ___________________ , First Name: _____________________

Circle Rec: M8am M4:30pm T10:30am W8am R10:30am R4:30pm F11:30am

4) (7 pts) With proof, find the smallest positive integer with **exactly 32 divisors**.

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5) (6 pts) With proof, find all possible remainders for the expression x^4 is divided by 5, given that x is an integer.

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6) (12 pts) Find all integer solutions (x, y) to the equation $232x + 105y = 14$.

COT 3100 Section 2 Exam #2 - Part 3 (Induction) - 35 pts (3/9/2023)

Last Name: ___________________ , First Name: _____________________

Circle Rec: M8am M4:30pm T10:30am W8am R10:30am R4:30pm F11:30am

7) (10 pts) Using induction on n, prove for all non-negative integers n, that $13 \mid (4^{3n} + 12(5^{2n}))$.

8) (12 pts) Let t_n be a sequence defined as follows:

$$t_0 = 2,\ t_1 = 13,\ t_n = 7t_{n-1} - 10t_{n-2} \text{ for all integers } n \geq 2.$$

Prove, **for all non-negative integers** n, that $t_n = 3(5^n) - 2^n$, via strong induction with 2 base cases.

9) (12 pts) Let F_n denote the n^{th} Fibonacci number. Specifically, $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1}+F_{n-2}$, for all integers $n \geq 2$.

Using induction on n, prove for **<u>all non-negative integers n</u>**, that $\sum_{i=1}^{2n+1} F_i F_{i+1} = (F_{2n+2})^2$.

10) (1 pt) Mountains from which mountain range can be found in the Rocky Mountain National Park?

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Scratch Page – Please clearly label any work on this page you would like graded.