COT 3100 Exam #2 Information

Tuesday, March 17 1:30 - 2:45 pm

Please come early.

TAs will have four sheets of the exam to pass out again. Here is how they will do it differently:

1. At 1:25 pm, four TAs will simultaneously start passing out exams, one starting at the top left, one at the top right, one at the bottom left and one at the bottom right. Each will be responsible for passing out a single sheet.

2. TAs will still go row by row, two going up and two going down, but students can start on the exam as soon as they receive a sheet.

3. Once two TAs finish with the left side and two TAs finish with the right side, all students should have 2 of the 4 sheets.

4. Then, the pairs of TAs will swap sides so students get the other 2 sheets.

5. At this point, if anyone is missing sheets, they need to come up front and get them.

Exam Format and Aids

There will be several free response questions totaling 100 points.

There will be no aids for this exam. Please memorize summation formulas, arithmetic sequence formulas and geometric sequence formulas. No calculators either, of course.

Outline of Intro to Discrete Exam #2 Topics

I. Number Theory (40% of the exam)
- **a. Division Algorithm**
- **b. Euclid's Algorithm**
- **c. Extended Euclid's Algorithm**
- **d. Full Solution to ax+by = c for integers given a,b,c.**
- **e. Finding modular inverses**
- **f. Divisibility proofs**
- **g. Pi notation**
- **h. Fundamental Thm of Algebra**
- **i. Least Common Multiple (LCM)**
- **j. Connection between LCM and GCD**
- **k. Calculating # of divisors of an integer.**
- **l. Calculating the sum of divisors of an integer.**
- **m. Calculating the number of times prime p divides into n!**
- **o. Proof that $\sqrt{2}$ is irrational (from proof techniques)**

II. Arith Geo Series, Sums, Matrices (10% of the exam)
- **a. Arithmetic Series - solving for terms, sums, etc.**
- **b. Geometric Series - solving for terms, sums, etc.**
- **c. How to recursively define sequences**
- **d. Definition of summation notation**
- **e. Summation Rules**
- **f. Telescopic sum idea**
- **g. Idea of bounding sums with integrals**
- **h. Matrix Addition, Subtraction, Multiplication**

III. Mathematical Induction (30% of the exam)
- **a. Base Case**
- **b. Inductive Hypothesis**
- **c. Inductive Step**
- **d. Summation Rules**
- **e. Not all induction problems use summations**

f. How to deal with inequalities
g. Strong Induction
h. Divisibility Problems
i. Matrix Exponentiation Problems
j. Problems with recursively defined sequences
k. Problems with Harmonic numbers
l. Unorthodox Examples - NIM, Nuggets, Trominos

IV. Counting (20% of the exam)
a. Addition Rule
b. Subtraction Rule
c. Multiplication Rule
d. Division Rule
e. Permutations
f. Permutations with repeated items
g. Combinations
h. Permutations with restrictions (no consecutive vowels, etc.)

Reading from Textbook

Number Theory: 4.1 - 4.4
Sums, Matrices: 2.4, 2.6
Mathematical Induction: 5.1 - 5.3
Counting: 6.1 - 6.3

What to Study

1) Read the notes.
2) Skim textbook.
3) Practice problems posted online in my archive.
4) Look over written lectures and past homework solutions.

Sample Questions

1) Determine 59^{-1} mod 203. Please express your answer as an integer in between 0 and 202. In order to earn full credit you must use the Extended Euclidean Algorithm.

2) Find the sum of divisors of 123,000 and express this in prime factorized form.

3) A geometric series has $a_3 = 54$ and $a_5 = 6$. What are the possible values of a_6?

4) Using induction on n, prove for all non-negative integers n that $9 \mid (2^{2n} + 6n - 1)$.

5) Using induction on n, prove for all positive integers n that $\sum_{i=1}^{n} i^2 \leq n^3$.

6) Let $H_n = \sum_{i=1}^{n} \frac{1}{i}$. Using induction on n, prove for all positive integers n that

$$\sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$$

7) Prove for all positive integers n, that in a two player NIM game with two piles of stones, the second player wins if and only if the number of stones in each pile is equal. (On a turn, a player must select a single pile and remove a positive number of stones from it. The winner is the player who removes the last stone.)

8) Let $M = \begin{bmatrix} a & 0 \\ 1 & 1 \end{bmatrix}$, where a is a given positive constant not equal to 1. Prove using induction on n, for all positive integers n, that $M^n = \begin{bmatrix} a^n & 0 \\ \frac{a^n - 1}{a - 1} & 1 \end{bmatrix}$.

9) Prove using strong induction on n with 3 base cases, that if and only if $3 \mid n$, then $2 \mid F_n$. where F_n represents the n^{th} Fibonacci number. (Note: For this question use the following $F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all integers $n \geq 2$.)

10) How many permutations are there of MISSISSIPPI? How many of those permutations don't have consecutive Is?

Note: Solutions to some of these were worked out in class and included in the daily scanned .pdfs of notes on the course grid of notes.