

Spring 2020 COT 3100 Exam #2 (3/17/2020) (Note: Out of 100 points) - Page 1, 2

Last Name: _____ , **First Name :** _____

Please circle your lab section

Lab Section: 11(T4:30) 12(R4:30) 13(R5:30) 14(F3:30)
15(F8:30) 16(F 9:30) 17(F 12:30)

1) (10 pts) On Day 1, a pond has 4^{20} particles of algae. (It's a very large pond...) Luckily, you have hired a top notch company to get rid of the algae. They guarantee that each day, they will cut the number of particles of algae in the pond by half. (So, on day 3, there are only one quarter the number of particles there were on day 1.) If the guarantee is accurate, on which day will there be exactly 1 particle of algae left in the pond?

2) (10 pts) An ordered triple of positive integers (a, b, c) is called a Pythagorean Triple if $a^2 + b^2 = c^2$. Prove that if m and n are positive integers with $m > n$, then $(m^2 - n^2, 2mn, m^2 + n^2)$ is a Pythagorean triple. Use this result to calculate 5 different Pythagorean Triples. (Credit will be given only if you use the result to derive the triples. No credit will be given if you just list triples you happen to know already.)

3) (15 pts) Find the set of all ordered pairs (x, y) of integers which satisfy the equation

$$386x + 170y = 14.$$

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4) (12 pts) Let $n = 6^{13}15^{20}$.

(a) (3 pts) What is the prime factorization of n ?

(b) (3 pts) How many divisors does n have? (**Leave answer as a product of 3 integers.**)

(c) (6 pts) How many divisors of n are perfect squares? (**Leave answer as a product of 3 integers.**)

5) (12 pts) Using induction on n , prove for all positive integers n that

$$\sum_{m=1}^n m(m!) = (n+1)! - 1$$

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6) (15 pts) Recall that the Fibonacci sequence is defined as follows: $F_0 = 0$, $F_1 = 1$, and for all $n > 1$, $F_n = F_{n-1} + F_{n-2}$. Using induction on n , prove for all positive integers n , that

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^n = \begin{bmatrix} F_{2n-1} & F_{2n} \\ F_{2n} & F_{2n+1} \end{bmatrix}$$

Note: for questions 7 through 10, please leave your answers in combinations, factorials, powers, products, sums and any reasonable combination thereof.

7) (2 pts) How many permutations of the word HOUSE are there?

8) (4 pts) How many permutations of the word FOOTBALL are there?

9) (10 pts) How many of the permutations of FOOTBALL do NOT have consecutive vowels?

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10) (8 pts) There are two lines of students. The first line has students s_1, s_2, s_3, s_4 and s_5 , in that order. The second line has students t_1, t_2, t_3, t_4, t_5 , and t_6 , in order. Both lines will be merged into one as follows: If both lines still have students in them, one of the two lines is randomly selected and the person in the front of that line will go to the back of the merged line. If only one line remains, then all of those students will join in the back of the merged line, in their original order. For example, one way in which the students could line up is: $s_1, t_1, t_2, t_3, s_2, t_4, t_5, s_3, t_6, s_4$ and s_5 . Using this procedure, how many possible ways could these 11 students line up?

11) (2 pts) What state is directly north of South Dakota? _____