

Spring 2017 COT 3100 Final Exam: Part B Solutions

Date: April 27, 2017

8) (12 pts - Logic) Use the rules of inference and laws of logic to show the following argument:

$p \rightarrow r$
 $q \rightarrow r$
 \bar{r}
 $\bar{s} \rightarrow p \vee q$

 s

Solution

- | | |
|-----------------------------------|--------------------------------------|
| 1. $p \rightarrow r$ | Premise |
| 2. \bar{r} | Premise |
| 3. \bar{p} | Modus Tollens with (1) and (2) |
| 4. $q \rightarrow r$ | Premise |
| 5. \bar{q} | Modus Tollens with (4) and (2) |
| 6. $\bar{p} \wedge \bar{q}$ | Rule of Conjunction with (3) and (5) |
| 7. $\overline{p \vee q}$ | De Morgan's Law with (6) |
| 8. $\bar{s} \rightarrow p \vee q$ | Premise |
| 9. $\bar{\bar{s}}$ | Modus Tollens with (8) and (7) |
| 10. s | Law of Double Negation |

Grading: 10 pts for steps, 2 pts for reasons. So correct response with no reasons get 10/12. For the reasons give 2/2 if all are accurate, 1/2 if some are accurate and at least one is wrong, 0/2 if missing. For steps, if most are there, subtract 1 pt per incorrect step. If most are missing, give 1 pt per reasonable step. So, they can earn 4 pts for just stating the premises.

9) (12 pts - Functions) Find the inverse of the following function:

$$f(x) = 3x^2 + 24x + 13, \text{ with a domain of all real } x, x \leq -4.$$

Solution

Switch x and y and solve for the new y:

$$\begin{aligned}x &= 3y^2 + 24y + 13 \\x &= 3(y^2 + 8y + 16) - 48 + 13 \\x &= 3(y + 4)^2 - 35 \\x + 35 &= 3(y + 4)^2 \\ \frac{x + 35}{3} &= (y + 4)^2 \\ \pm \sqrt{\frac{x + 35}{3}} &= y + 4 \\ y &= -4 \pm \sqrt{\frac{x + 35}{3}}\end{aligned}$$

If the original domain is all real $x \leq -4$, then the range of the inverse function will be all real values of y with $y \leq -4$. For this to be the range, we must choose the minus sign between the two signs. Thus, the correct inverse function is:

$$f^{-1}(x) = -4 - \sqrt{\frac{x + 35}{3}}$$

Grading: 2 pts switching x and y or equivalent, 4 pts completing the square, 2 pts for square root, 2 pts for solving for y, 2 pts for choosing the minus sign.

10) (1 pt - Just for Fun) After what former mayor of Orlando is Bill Frederick Park named?

Bill Frederick (give to all)