Spring 2017 COT 3100 Exam #*1* Multiple Choice
Logic, Sets, Number Theory

Version C

Please double check that you bubbled your PID and exam version on your scantron. If you do not, you will get a ZERO on this exam.

1) Let a and b be positive integers such that a < b, gcd(a, b) = 6, and lcm(a, b) = 360. What is the minimum possible value of a?

a) 1 **b) 6** c) 12 d) 30 e) None of the Above

Solution
Answer: B
a can not be smaller than gcd(a,b) and the smallest possible number for it will be 6. In that case b will be 360.

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2) How many divisors does $2^3 3^3 97^4$ have?

a) 10 b) 36 c) 72 d) 108 **e) None of the Above**

Solution
Answer: E
For any number in form of $a^x b^y$ the number of divisors will be (x+1)(y+1) because we can select 0,1,2,...or x+1 as a power for a and select 0,1,2,...or y+1 as a power for b.
Then we have (3+1) cases for power of the 2, (3+1) cases for power of the 3
and (4+1) cases for power of the 97. So all the number of the divisors will be 4X4X5= 80

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3) How many integer solutions for x and y does 26x + 91y = 1301 have?

a) 1 b) 13 c) 26 d) infinite e) None of the Above

Solution
Answer: E
Notice that 13 divides evenly into both 26 and 91, but it doesn't divide evenly into 1301. It follows that there are no solutions.

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4) What is the remainder when 103^{101} is divided by 101?

a) 1 **b) 2** c) 16 d) 103 e) None of the Above

Solution

Answer: B

Using Fermat rule $103^{100} \equiv 1(\text{mod } 101)$ then $103^{101} \equiv 103 \equiv 2(\text{mod } 101)$

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5) Consider the following argument: "John either owns a yacht or a bicycle. John does not own a yacht. We can conclude that John owns a bicycle." Which rule of inference is being used in deriving the conclusion?

a) Rule of Conjunctive Simplification b) Rule of Conjunction

c) Rule of Disjunctive Syllogism d) Modus Tollens e) None of the Above

Solution

Answer: C

By rule of disjunctive syllogism if we have p or q, and not q, we can conclude p.

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6) Consider the following true statement over the real numbers: $\exists x \forall y [xy = 0]$. Which of the following values can we set x equal to, to prove that this statement is true?

a) y b) 10^6 c) 1 **d) 0** e) None of the Above

Solution

Answer: D

There exists x = 0 that for all y, (0)y = 0

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7) Which of the following is the contrapositive of the Boolean expression $\bar{r} \to (\bar{p} \vee q)$.

a) $(p \wedge \bar{q}) \to r$ b) $(p \vee \bar{q}) \to \bar{r}$ c) $(p \wedge \bar{q}) \to \bar{r}$

d) $(\overline{p \wedge \bar{q}}) \to r$ e) None of the Above

Solution

Answer: A

We know that contrapositive of $r \to q$ is $\bar{q} \to \bar{r}$. Now, let's consider simplifying $\overline{\bar{p} \vee q}$. We must apply DeMorgan's getting $\bar{\bar{p}} \wedge \bar{q}$. Then using Double Negation, we get $p \wedge \bar{q}$. Finally, the negation of $\bar{r}$ is just r.

8) Let $P(x, y) = "x + y = 0"$. The statement $\forall x \exists y [P(x, y)]$ is a true statement. To prove that this statement is true, for any arbitrary value of x, we must find a value of y which makes the statement true. What is the value of y that makes the statement true?

a) -x b) x c) 0 d) 1 e) None of the Above

Solution

Answer: A

For all x, there exists y = -x that P = x + y = x - x = 0

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9) Let A, B and C be sets with A = {1, 2, 3, 5}, B = {2, 4, 7, 8} and C = {1, 2, 6, 8}. How many elements does AXB - CXC contain?

a) 0 **b) 12** c) 14 d) 16 e) None of the Above

Solution

Answer: B

Number of elements of |A|×|B|= 4× 4 =16

Number of elements of |C|×|C|= 4× 4 =16

A×B ∩ C × C = {(1,2), (2,2), (1, 8), (2, 8)}

| A×B ∩ C × C|= 4

$|A \times B - C \times C|$= $|A \times B|$-| A×B ∩ C × C| = 16-4 = 12

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10) Let A and B be sets such that $|A \cup B|$ = 49 and $|A \cap B|$ = 11. If A has twice as many elements as B, how many elements does A have?

a) 11 b) 19 c) 20 **d) 40** e) None of the Above

Solution

Answer: D

From cardinality of the set union we know that:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

And also from given data we know $|A| = 2|B|$. So, we will have:

$$|A \cup B| = |A| + |B| - |A \cap B| => 49 = 2|B| + |B| - 11 => |B| = 20$$

|A| = 2 |B| = 40

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11) Let the set A = {1, 3, 4, 5, 7, 10}. How many elements of $\wp(A)$ contain in between one and two odd numbers, inclusive, and no even numbers?

a) 4 b) 6 **c) 10** d) 14 e) None of the Above

Solution
Answer: C

The ten desired subsets are {1},{3},{5},{7},{1,3},{1,5},{1,7},{3,5},{3,7},and {5,7}.

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12) Define a set s(A, B, C), in terms of sets A, B and C as follows:

$$s(A,B,C) = \{x | ((x \in A) \vee (x \in B) \vee (x \in C)) \wedge (x \notin A \cap B) \wedge (x \notin A \cap C) \wedge (x \notin B \cap C)\}.$$

Let A = {1, 2, 5, 9, 12, 17}, B = {2, 3, 9, 15, 20}, and C = {3, 8, 17, 19, 20, 21, 24}. What is $s(A,B,C)$?

a) {1, 2, 3, 5, 8, 9, 12, 15, 17, 19, 20, 21, 24} b) {4, 6, 7, 10, 11, 13, 14, 16, 18, 22, 23}

c) {1, 5, 8, 12, 15, 19, 21, 24} d) {2, 3, 9, 17, 20} e) None of the Above

Solution
Answer: C
s(A,B,C) should not contain 2,3,9,17, and 20, the elements that are in 2 or more of the three sets and all the elements should be in A or B or C. (s(A,B,C) contains the elements that are in precisely one of the three sets A, B or C.)

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13) Bruno Mars has won Grammy Awards in the past for his songs "Uptown Funk" and "Just The Way You Are". This year, he won a Grammy as a producer for the Album 25, by Adele. With what planet in our solar system does he share his last name?

a) Earth b) Jupiter **c) Mars** d) Saturn e) None of the Above

Solution
Answer: C
