Spring 2017 COT 3100 Exam #*1* Multiple Choice
Logic, Sets, Number Theory

VERSION B

<u>Please double check that you bubbled your PID and exam version on your scantron. If you do not, you will get a ZERO on this exam.</u>

1) Let A, B and C be sets with A = {1, 2, 3, 5}, B = {2, 4, 7, 8} and C = {1, 2, 6, 8}. How many elements does $A \times B - C \times C$ contain?

a) 0 b) 2 c) 10 d) 12 e) None of the Above

<u>Solution</u>

Answer: D

Number of elements of |A|×|B|= 4× 4 =16

Number of elements of |C|×|C|= 4× 4 =16

A×B ∩ C × C = {(1,2), (2,2), (1, 8), (2, 8)}

| A×B ∩ C × C|= 4

$|A \times B - C \times C|$= $|A \times B|$-| A×B ∩ C × C| = 16-4 = 12

2) Let A and B be sets such that $|A \cup B|$ = 46 and $|A \cap B|$ = 11. If A has twice as many elements as B, how many elements does A have?

a) 11 b) 19 c) 38 d) 57 e) None of the Above

<u>Solution</u>

Answer: C

From cardinality of the set union we know that:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

And also from given data we know $|A| = 2|B|$. So, we will have:

$$|A \cup B| = |A| + |B| - |A \cap B| => 46 = 2|B| + |B| - 11 => |B| = 19$$

|A| = 2 |B| =38

3) Let the set A = {1, 3, 4, 5, 7, 10}. How many elements of $\wp(A)$ contain in between one and three odd numbers, inclusive, and no even numbers?

a) 14 b) 15 c) 16 d) 64 e) None of the Above

Solution

Answer: A

We should pick 1 or 2 or 3 number from this set: {1,3,5,7}. So, totally we will have

$$\binom{4}{1} + \binom{4}{2} + \binom{4}{3} = 4 + 6 + 4 = 14$$

Note: since combinations have not been taught in the class yet, the intended solution was to list out each possible combination: {1}, {3}, {5}, {7}, {1,3}, {1,5}, {1,7}, {3,5}, {3,7}, {5,7}, {1,3,5}, {1,3,7}, {1,5,7}, and {3,5,7}.

4) Define a set s(A, B, C), in terms of sets A, B and C as follows:

$s(A, B, C) = \{x | ((x \in A) \vee (x \in B) \vee (x \in C)) \wedge (x \notin A \cap B) \wedge (x \notin A \cap C) \wedge (x \notin B \cap C)\}$.

Let A = {1, 2, 5, 9, 12, 17}, B = {2, 3, 9, 15, 20}, and C = {3, 8, 17, 19, 20, 21, 24}. What is $s(A, B, C)$?

a) {1, 2, 3, 5, 8, 9, 12, 15, 17, 19, 20, 21, 24} b) {4, 6, 7, 10, 11, 13, 14, 16, 18, 22, 23}

c) {2, 3, 9, 17, 20} d) {1, 5, 8, 12, 15, 19, 21, 24} e) None of the Above

Solution

Answer: D

s(A,B,C) should not contain 2,3,9,17, and 20, the elements that are in 2 or more of the three sets and all the elements should be in A or B or C. (s(A,B,C) contains the elements that are in precisely one of the three sets A, B or C.)

5) Let a and b be positive integers such that a < b, gcd(a, b) = 6, and lcm(a, b) = 360. What is the minimum possible value of a?

a) 6 b) 12 c) 30 d) 60 e) None of the Above

Solution

Answer: A

a must be at least 6 and if we set b to be 360, we see that a = 6, b = 360 satisfies the given constraints so 6 is the correct answer.

6) How many divisors does $2^3 3^3 97^2$ have?

a) 8 b) 18 c) 48 d) 108 e) None of the Above

Solution

Answer: C

Number of divisors is: (3+1)*(3+1)*(2+1) = 4*4*3= 48

7) How many integer solutions for x and y does 26x + 91y = 13 have?

a) 0 b) 1 c) 13 d) infinite e) None of the Above

Solution

Answer: D

Because gcd(26,91) is equal to 1, so according to Euclidean algorithm it will have at least one answer. Then, from that one answer we can find infinite answers by adding an offset to x and subtracting an offset from y. (These offsets are 7 and 2, respectively.)

8) What is the remainder when 103^{104} is divided by 101?

a) 1 b) 2 c) 16 d) 103 e) None of the Above

Solution

Answer: C

We can use Fermat rule to write following equation:

$$2^{100} \equiv 1 \ mod(101)$$

multiply both side of equation by 2^4: $2^{104} \equiv 2^4 \ mod(101)$

then we can replace 2 with 103: $103^{104} \equiv 16 \ mod(101)$

9) Consider the following argument: "John either owns a yacht or a bicycle. John does not own a yacht. We can conclude that John owns a bicycle." Which rule of inference is being used in deriving the conclusion?

a) Rule of Conjunction b) Rule of Conjunctive Simplification

c) Rule of Disjunctive Syllogism d) Modus Tollens e) None of the Above

Solution

Answer: C

It is same as we say p is having yacht and q is having bicycle (p∨q) and we know that p is false $((p \vee q) \wedge \bar{p}) \rightarrow q)$. So, from Disjunctive Syllogism we can conclude that q is true.

10) Consider the following true statement over the real numbers: $\exists x \forall y [xy = 0]$. Which of the following values can we set x equal to, to prove that this statement is true?

a) 1　　b) 10^6　　c) y　　d) y^2　　e) None of the Above

Solution

Answer: E

There is one vaule of x, x=0, that makes xy=0, no matter what y is.

11) Which of the following is the contrapositive of the Boolean expression $\bar{r} \to (\bar{p} \vee q)$.

a) $(p \vee \bar{q}) \to r$　　b) $(p \vee \bar{q}) \to \bar{r}$　　c) $(p \wedge \bar{q}) \to \bar{r}$

d) $(\overline{p \wedge \bar{q}}) \to r$　　e) None of the Above

Solution

Answer: E

We know that contrapositive of $r \to q$ is $\bar{q} \to \bar{r}$. Now, let's consider simplifying $\overline{\bar{p} \vee q}$. We must apply DeMorgan's getting $\bar{\bar{p}} \wedge \bar{q}$. Then using Double Negation, we get $p \wedge \bar{q}$. Finally, the negation of $\bar{r}$ is just r. Thus, the desired answer is $(p \wedge \bar{q}) \to r$

12) Let $P(x, y) =$ "$x + y = 0$". The statement $\forall x \exists y [P(x, y)]$ is a true statement. To prove that this statement is true, for any arbitrary value of x, we must find a value of y which makes the statement true. What is the value of y that makes the statement true?

a) 0　　b) 1　　c) -x　　d) x　　e) None of the Above

Solution

Answer: C

For all the x, there is at least one y=-x which make statement true.

13) Bruno Mars has won Grammy Awards in the past for his songs "Uptown Funk" and "Just The Way You Are". This year, he won a Grammy as a producer for the Album 25, by Adele. With what planet in our solar system does he share his last name?

a) Earth　　b) Mars　　c) Jupiter　　d) Saturn　　e) None of the Above

Solution

Answer: B