Spring 2017 COT 3100 Exam #*1* Multiple Choice
Logic, Sets, Number Theory

VERSION A

<u>Please double check that you bubbled your PID and exam version on your scantron. If you do not, you will get a ZERO on this exam.</u>

1) Consider the following argument: "John either owns a yacht or a bicycle. John does not own a yacht. We can conclude that John owns a bicycle." Which rule of inference is being used in deriving the conclusion?

a) Rule of Conjunction   b) Rule of Disjunctive Syllogism

c) Rule of Conjunctive Simplification   d) Modus Tollens   e) None of the Above

<u>Solution</u>

Answer: B

Because by rule of disjunctive syllogism if we have p or q, and not q, we can conclude p.

2) Consider the following true statement over the real numbers: $\exists x \forall y [xy = 0]$. Which of the following values can we set x equal to, to prove that this statement is true?

a) 0   b) 1   c) y   d) y^2 e) None of the Above

<u>Solution</u>

Answer: A

Since we want to show there exist a x which makes xy=0.

3) Which of the following is the contrapositive of the Boolean expression $\bar{r} \rightarrow (\bar{p} \vee q)$.

a) $(p \vee \bar{q}) \rightarrow r$ b) $(p \vee \bar{q}) \rightarrow \bar{r}$ c) $(p \wedge \bar{q}) \rightarrow \bar{r}$

d) $(p \wedge \bar{q}) \rightarrow r$ e) None of the Above

<u>Solution</u>

Answer: D

We know that contrapositive of $r \rightarrow q$ is $\bar{q} \rightarrow \bar{r}$. Now, let's consider simplifying $\overline{\bar{p} \vee q}$. We must apply DeMorgan's getting $\bar{\bar{p}} \wedge \bar{q}$. Then using Double Negation, we get $p \wedge \bar{q}$. Finally, the negation of $\bar{r}$ is just r.

4) Let $P(x, y) =$ "$x + y = 0$". The statement $\forall x \exists y[P(x, y)]$ is a true statement. To prove that this statement is true, for any arbitrary value of x, we must find a value of y which makes the statement true. What is the value of y that makes the statement true?

a) 0 b) 1 c) x d) x^2 e) None of the Above

Solution

Answer: E

The value of y should be -x to make x+y=0

5) Let A, B and C be sets with A = {1, 3, 5}, B = {2, 4, 7, 8} and C = {1, 2, 6, 8}. How many elements does $A \times B - C \times C$ contain?

a) 0 b) 2 c) 10 d) 12 e) None of the Above

Solution

Answer: C

$A \times B - C \times C$ = {(1,4),(1,7),(3,2),(3,4),(3,7),(3,8),(5,2),(5,4),(5,7),(5,8)}

6) Let A and B be sets such that $|A \cup B|$ = 46 and $|A \cap B|$ = 11. If A has twice as many elements as B, how many elements does B have?

a) 11 b) 19 c) 38 d) 57 e) None of the Above

Solution

Answer: B

$|A \cup B|+|A \cap B|=|A|+|B| = 2|B| + |B|=46+11 \quad 3|B| = 57 \quad |B| = 19$

7) Let the set A = {1, 3, 4, 5, 7, 10}. How many elements of $\wp(A)$ contain in between one and three odd numbers, inclusive, and no even numbers?

a) 4 b) 15 c) 16 d) 64 e) None of the Above

Solution

Answer: E

{1},{3},{5},{7},{1,3},{1,5},{1,7},{3,5},{3,7},{5,7},{1,3,5},{3,5,7},{1,5,7},{1,3,7}. Answer is 14

8) Define a set s(A, B, C), in terms of sets A, B and C as follows:

$$s(A,B,C) = \{x | ((x \in A) \vee (x \in B) \vee (x \in C)) \wedge (x \notin A \cap B) \wedge (x \notin A \cap C) \wedge (x \notin B \cap C)\}.$$

Let A = {1, 2, 5, 9, 12, 17}, B = {2, 3, 9, 15, 20}, and C = {3, 8, 17, 19, 20, 21, 24}. What is $s(A,B,C)$?

a) {1, 2, 3, 5, 8, 9, 12, 15, 17, 19, 20, 21, 24}　　b) {4, 6, 7, 10, 11, 13, 14, 16, 18, 22, 23}

c) {1, 5, 8, 12, 15, 19, 21, 24}　　d) {2, 3, 9, 17, 20}　　e) None of the Above

Solution

Answer: C

s(A,B,C) should not contain 2,3,9,17, and 20, the elements that are in 2 or more of the three sets and all the elements should be in A or B or C. (s(A,B,C) contains the elements that are in precisely one of the three sets A, B or C.)

9) Let a and b be positive integers such that a < b, gcd(a, b) = 6, and lcm(a, b) = 360. What is the maximum possible value of a?

a) 6　　b) 12　　c) 30　　d) 60　　e) None of the Above

Solution

Answer: C

Since the gcd is 6, we must have a = 6x, b = 6y, where gcd(x, y) = 1. Furthermore, we have ab = 6 x 360, so xy = 60.Thus, we are looking for different pairs of values (x, y), where x < y, gcd(x,y)=1 and x is maximal. The ordered pairs we must consider are (1, 60), (2, 30), (3, 20), (4, 15), (5, 12), and (6, 10). But of these, we must remove (2, 30) and (6, 10) from consideration since both values in the pairs share a common factor of 2. It follows that the maximal value of x is 5 and a = 30 is the largest possible value of a which is consistent with the given information.

10) How many divisors does $2^3 3^8 97^2$ have?

a) 13　　b) 16　　c) 48　　d) 108　　e) None of the Above

Solution

Answer: D

The number of divisors for numbers in form of $a^x b^y$ will be (x+1)(y+1) so (3+1)(8+1)(2+1)=108

11) How many integer solutions for x and y does $26x + 91y = 1013$ have?

a) 0 b) 1 c) 13 d) infinite e) None of the Above

Solution
Answer: A
Notice that 13 divides evenly into both 26 and 91, but it doesn't divide evenly into 1013. It follows that there are no solutions.

12) What is the remainder when 103^{105} is divided by 101?

a) 1 b) 2 c) 16 d) 103 e) None of the Above

Solution
Answer: E
Using Fermat's rule $103^{100} \equiv 1 \pmod{101}$ then $103^{105} \equiv 103^{100} \equiv 103^5 \equiv 2^5 \equiv 32 \pmod{101}$.

13) Bruno Mars has won Grammy Awards in the past for his songs "Uptown Funk" and "Just The Way You Are". This year, he won a Grammy as a producer for the Album 25, by Adele. With what planet in our solar system does he share his last name?

a) Mars b) Earth c) Jupiter d) Saturn e) None of the Above

Solution
Answer: A