Spring 2014 COT 3100 HW5 Solutions (by Payal Jotwani)

1) (5pts) Determine the prime factorizations of the following integers:

a) 46008 = $2^3 * 3^4 * 71$ **(1 pt)**
b) 54839025 = $3^7 * 5^2 * 17^1 * 59^1$ **(1 pt)**
c) 31659 = $3^1 * 61^1 * 173^1$ **(1 pt)**
d) 1013 = 1013^1 **(1 pt)**
e) 396842322 = $2^1 * 3^1 * 17^1 * 19^1 * 23^1 * 29^1 * 307^1$ **(1 pt)**

Note: These factorizations were obtained with the attached program primefact.java.

Grading: Only take off 1 pt total if there's no indication as to how the factorizations were arrived at.

2) (15 pts) Find the number of divisors of each of the questions in #1 as well as the sum of those divisors.

a) n = 46008
numD (n) = (3+1)*(4+1)*(1+1) = $2^3 * 5^1$= 40 **(1 pt)**
sumD(n) = $\frac{2^4-1}{2-1} * \frac{3^5-1}{3-1} * \frac{71^2-1}{71-1}$ = 15 * 121 * 72
= 130680 **(2 pt)**

b) n = 54839025
numD(n) = (7+1)*(2+1)*(1+1)*(1+1) = $2^5 * 3^1$ = 96 **(1 pt)**
sumD(n) = $\frac{3^8-1}{3-1} * \frac{5^3-1}{5-1} * \frac{17^2-1}{17-1} * \frac{59^2-1}{59-1}$
= 3280 * 31 * 18 * 60
= 109814400 **(2 pt)**

c) n = 31659
numD(n) = (1+1)*(1+1)*(1+1) = 2^3 **(1 pt)**
sumD(n) = $\frac{3^2-1}{3-1} * \frac{61^2-1}{61-1} * \frac{173^2-1}{173-1}$
= 4 * 62 * 174
= 43152 **(2 pt)**

d) n = 1013
numD(n)= (1+1) = 2^1 **(1 pt)**
sumD(n) = $\frac{1013^2-1}{1013-1}$ = 1014 **(2 pt)**

e) n = 396842322
numD(n) = (1+1) * (1+1) * (1+1) * (1+1)*(1+1)*(1+1) *(1+1) = 2^7 = 128 **(1 pt)**
sumD(n) = $\frac{2^2-1}{2-1} * \frac{3^2-1}{3-1} * \frac{17^2-1}{17-1} * \frac{19^2-1}{19-1} * \frac{23^2-1}{23-1} * \frac{29^2-1}{29-1}$
$* \frac{307^2-1}{307-1}$ = 3 * 4 * 18 * 20 * 24 * 30 * 307 = 958003200 **(2 pts)**

3) (10 pts) Run the Sieve of Eratosthenes for the integers 1 through 121, with the following modifications:

1) Stop the outer loop after crossing off multiples of 11, since every composite integers less than or equal to 121 has at least one multiple less than equal to 11.

2) If a number you arrive at is already crossed off, do not cross of its multiples again, just advance to the next number.

Count the number of times a value is crossed off in running the algorithm. (For example, on the first iteration, 2 is circled and all of its multiples, from 4 to 120 are crossed off. This counts as 59 cross offs.)

The only primes less than or equal to 11 are 2, 3, 5, 7 and 11. Thus, all of our cross offs will occur when the outer loop variable is equal to these five values. As already discussed in the question, there are 59 cross offs due to 2. In general, for an arbitarary

integer k, when we are checking primality upto n, we would cross off n/k - 1 items, where we define division to be integer division. (In math, we'd typically use the floor function, but for some bizarre reason, the symbol for the floor function isn't working on my equation editor and I need to get these solutions done!) The reason is as follows, for every *full* group of k integers, we cross off the last one. However many full groups appear in the positive integers upto n is how many multiples of k there are. Of these multiples, k itself, doesn't get crossed off when k is prime. This is why there is the minus one.

Now that we have a general formula, we can quickly calculate the number of cross offs as follows: 121/2 - 1 + 121/3 - 1 + 121/5 - 1 + 121/7 - 1 + 121/11 - 1 = 147.

Grading: For analytical technique above: 6 pts for general formula for one prime, 4 pts for answer, only 1 pt off for off by one type error. 3 pts off for NOT counting more than one cross off for the same value. (For example 6 gets crossed off twice, not once.)

For brute force by hand: 3 pts off for NOT counting more than one cross off for the same value. Otherwise, 10 pts if the answer is correct, 9 pts if its within 5, 7 pts if they made one systemic error that caused a greater error. If it's worse than that, use your discretion for partial credit.

For a program: 6 pts for any reasonable system used in the code, 3 pts for allowing a number to get crossed off more than once, 1 pt for correct final answer.

4) (10 pts) Find the following greatest common divisors using Euclid's Algorithm. **Note: NO CREDIT WILL BE GIVEN IF YOU USE ANOTHER METHOD.**

a) gcd(374, 191)

374	=	1 x 191	+	183
191	=	1 x 183	+	8
183	=	22 x 8	+	7
8	=	1 x 7	+	**1**
7	=	7 x 1	+	0

Ans: 1

(2 pts for steps, 1 pt for correct answer)

b) gcd(532, 189)

532	=	2 x 189	+	154
189	=	1 x 154	+	35
154	=	4 x 35	+	14
35	=	2 x 14	+	**7**
14	=	2 x 7	+	0

Ans: 7

(2 pts for steps, 1 pt for correct answer)

c) gcd(233, 144)

233	=	1 x 144	+	89
144	=	1 x 89	+	55
89	=	1 x 55	+	34
55	=	1 x 34	+	21
34	=	1 x 21	+	13
21	=	1 x 13	+	8
13	=	1 x 8	+	5
8	=	1 x 5	+	3
5	=	1 x 3	+	2
3	=	1 x 2	+	**1**
2	=	2 x 1	+	0

Ans: 1

(3 pts for steps, 1 pt for correct answer)

5) (10 pts) Using the Extended Euclidean Algorithm, find all sets of integers a and b which satisfy the following equation: 374a + 191b = 1.

Bigger Number = 374
Smaller Number = 191

374	=	1 x 191	+	183	
191	=	1 x 183	+	8	
183	=	22 x 8	+	7	
8	=	1 x 7	+	**1**	
7	=	7 x 1	+	0	**(2 pts)**

1 x 374	-	1 x 191	=	183	equation 1
1 x 191	-	1 x 183	=	8	equation 2
1 x 183	-	22 x 8	=	7	equation 3
1 x 8	-	1 x 7	=	**1**	equation 4

Substituting for 7 in equation 4, as per equation 3 we get,

1 x 8 - 1 x (1 x 183 – 22 x 8) = **1**

=> - 1 x 183 + 23 x 8 = **1** equation 5 **(2 pts)**

Substituting for 8 in equation 5, as per equation 2 we get,

-1 x 183 + 23 x (1 x 191 - 1 x 183) = 1
=> -24 x 183 + 23 x 191 = 1 equation 6 **(2 pts)**

Substituting for 183 in equation 6, as per equation 1 we get,
-24 x (1 x 374 - 1 x 191) + 23 x 191 = 1
=> -24 x 374 + 24 x 191 + 23 x 191 = 1
=>-24 x 374 + 47 x 191 = 1 equation 7 **(2 pts)**

Therefore,
a = -24
& b = 47

(1 pt)

Dividing a and b by gcd, we get

374a + 191b = 0
=>374a = - 191b

a=191
b=-374

Then,
a= -24 + 191n
b= 47 – 374n , for n$\in Z$ **(1 pt)**