Spring 2014 COT 3100 Homework #3 Solutions (by Ashkan Paya)

1) g(x) = Number of flights x ever had, where x is a person. In the following example we have five people, Dave who has taken 12 flights, Asha who has taken 23, Maria who has taken 53, Demetrius who has taken 1, and Mary who has taken 5:

g(Dave) = 12, g(Asha) = 23, g(Maria) = 53, g(Demetrius) = 1, g(Mary) = 5

Grading: 5 pts, use your discretion

2) To prove that the given function is injective, we must show that if f(a) = f(b), then a = b. **(1 pt)** We use direct proof:

Assume f(a) = f(b). Plugging in for the function, we have:

2a = 2b, divide by 2 to get **(1 pt)**
a = b, as desired. **(1 pt)**

To prove that the function isn't surjective, consider the value y = 1. **(1 pt)** We must show that there is no x in the domain such that f(x) = 1. We know that if 2x = 1, then the only value of x that satisfies the equation is x = 0.5. But, this value is NOT in the domain of the function. **(1 pt)** Thus, for the given domain, no value of x satisfies the equation f(x) = 1. It follows that the function isn't a surjection.

3. $f(x) = (x-3)^2 - 5$

f^{-1}: $x = (y-3)^2 - 5 \Rightarrow x+5 = (y-3)^2 \Rightarrow f^{-1}$: $y = \sqrt{x+5} + 3$ (we do not need to consider negative square root because range of f^{-1} is domain of f which does not have negative numbers) **(3 pts – 1 pt for each step)**

Domain of f^{-1} (x)= Range of f(x) => $[-5, \infty)$. We determine this range by finding the minimum value of the function f on the given domain, which occurs at x = 3, where f(3) = -5, since perfect squares such as $(x-3)^2$ are always non-negative. **(2 pts total – 1 pt for each boundary)**

4. $f(x) = (2x+3)^2$, $g(x) = e^x$

- $f(g(x)) = (2e^x+3)^2$ **(2 pts)**
- $g(f(x)) = e^{(2x+3)^2}$ **(3 pts)**

5. $S = \frac{n(2a_1+(n-1)d)}{2} = \frac{40(2\times 7+39\times 4)}{2} = 3400$ **(2 pts formula, 2 pts plug in, 1 pt ans)**

6. $S = \frac{a_1(1-r^n)}{1-r} = \frac{3(1-2^{20})}{1-2} = 3145725$ **(2 pts formula, 2 pts plug in, 1 pt ans)**

7. $S = \frac{a_1}{1-r} = \frac{6}{1-\frac{1}{3}} = \frac{6}{\frac{2}{3}} = \frac{18}{2} = 9$ **(2 pts formula, 2 pts plug in, 1 pt ans)**

8. $\sum_1^{2n-1} 3i^2 + \sum_1^{2n-1} 2 = 3 * \frac{(2n-1)(2n)(4n-1)}{6} + 2(2n-1) = \frac{(2n-1)[(2n)(4n-1)+4]}{2}$
Grading: 1 pt split, 2 pts sum i², 1 pt sum 2, 1 pt ans

9. $\begin{bmatrix} -24 & 34 & 41 \\ 10 & 1 & 31 \\ -28 & 21 & -12 \end{bmatrix}$ (**Grading – ½ pt per correct entry, round up)**

10. $\begin{bmatrix} 3 & 4 & -1 \\ 2 & -5 & 4 \\ 1 & 2 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 41 \\ -24 \end{bmatrix}$ **(3 pts for 3x3 matrix, 1 pt for 2nd matrix, 1 pt for 3rd matrix)**