Spring 2014 COT 3100 HW1 Solutions by Ryan Zimmerman

1.

p	q	r	~q	~q v r	p -> (~q v r)
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	0	1
0	1	1	0	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	0	1	1

Grading = 4 pts total, ½ off per incorrect row, round down

2.

p	q	r	~p	~p <-> q	r -> p	(~p<->q) -> (r->p)
0	0	0	1	0	1	1
0	0	1	1	0	0	1
0	1	0	1	1	1	1
0	1	1	1	1	0	0
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	0	0	1	1
1	1	1	0	0	1	1

Grading = 4 pts total, ½ off per incorrect row, round down

3.

J = Jasmine attends and is happy
K = Kanti attends and is happy
S = Samir attends and is happy
A = J -> ~S
B = S -> K
C = K -> J

J	K	S	A	B	C	A^B^C
0	0	0	1	1	1	1
0	0	1	1	0	1	0
0	1	0	1	1	0	0
0	1	1	1	1	0	0
1	0	0	1	1	1	1
1	0	1	0	0	1	0
1	1	0	1	1	1	1
1	1	1	0	1	1	0

So, there are three possible combinations: None of them, Jasmine alone, or Jasmine and Kanti. **Grading = 4 pts total, ½ off per incorrect row, round down**

4.

(p->q) ^ (p->r)	Given
~~((p->q) ^ (p->r))	Double negation
~(~(p->q) v ~(p->r))	DeMorgan's law
~(~(~p v q) v ~(~p v r))	Implication identity
~((p ^ ~q) v (p ^ ~r))	DeMorgan's law
~(p ^ (~q v ~r))	Distributive law
~p v (q ^ r)	DeMorgan's law
p -> (q ^ r)	Implication identity

Grading = 5 pts total, 1 pt off for incorrect step, cap at 5

5.

((p v ~p) ^ (q v ~(~q v r))) v ((p v ~p)^q)	Given
((p v ~p) ^ (q v ~(~q v r) v q)	Distributive law
T ^ (q v ~(~q v r) v q)	Negation law
(q v ~(~q v r) v q)	Identity law
(q v q) v ~(~q v r)	Associative Law
q v ~(~q v r)	Idempotent law
q v (q ^ ~r)	DeMorgan's law
q	Absorption law

Grading = 5 pts total, 1 pt off for incorrect step, cap at 5

6.

a) for all x there exists some y and z | x= yz

let y = 1 and z = x for any x: x = (1)(x),

therefore the statement is true

b) there exists some x for any y | xy = 0

let x = 0: (0)(y) = 0,

therefore the statement is true

c) there exists some x for any y | xy = 1

xy = 1 -> x = 1/y, let y = 2, x = ½ , ½ is not an integer

therefore the statement is false, since no integer times 2 is 1.

d) for all x (x >0) -> (x^2 + x -6 > 0)

let x =1, 1^2 + 1 -6 = -4 < 0,

therefore the statement is false

e) there exists some x | (x > 2) ^ (x^2+x-6 < 0)

let f(x) = x^2 + x -6, f(2) = 2^2 + 2 – 6 = 0 and when x >2, f(x) increases therefore the statement is false. We can see this by nothing that for all positive x, the quantity x^2 + x is increasing, so the whole function must be as well. Technically, the function is increasing from x > -0.5.

Grading – 1 pt for parts a through d, 2 pts for part e. To get the point, "most" of the response (both answer and reasoning) has to be correct. For e, 1 pt for T/F, 1 pt for the justification.

7.

p -> t	Given
p	Given
t	Law of detachment (1, 2)
t -> q	Given
q	Law of detachment (3,4)
~q v ~s	Given
~s	Disjunctive syllogism (5,6)
r -> s	Given
~r	Law of contraposition (7,8)

Grading = 5 pts total, 1 pt off for incorrect step, cap at 5

8.

s -> t	Given
~t	Given
~s	Law of contraposition (1,2)
(~p v ~q) -> (s ^ r)	Given
~(s ^ r)	Domination law (3,4)
~(~p v ~q)	Law of contraposition (3,5)
p ^ q	DeMorgan's law (6)
p	Simplification (7)

Grading = 5 pts total, 1 pt off for incorrect step, cap at 5

9.

Prove: 3a+7 is even -> a is odd | a is an integer

Assume the opposite of what we want to prove, namely that a is even.
In this case, we can find an integer n such that a = 2n. **(2 pts)**
It follows that

3a + 7 = 3(2n) + 7 = 6n + 7 = 6n+6 + 1 = 2(3n+3) + 1 **(3 pts)**
Thus, 3a+7 is odd, since we've written it as 2 times an integer plus 1.
This contradicts our given information that 3a+7 is even.

It follows that our initial assumption that a is even is incorrect. We conclude (as desired) that a is odd. **(1 pt)**

Grading = 6 pts total allocated above.

10.
Prove: a is odd -> 3a+7 is even | a is an integer

Assume:

a is odd **(1 pt)**

Let:

a = 2n+1 | n is an integer **(1 pt)**

3a+7 = 3(2n+1) + 7
= 6n + 3 +7
= 6n + 10
= 2(3n+5) **(3 pts)**

This proves that 3a + 7 is even since we've expressed it as 2 times an integer.

Since, if 3a+7 is even, then a is odd and if a is odd, 3a+7 is even whenever a is an integer (see proofs in questions 9 and 10), a is odd iff 3a+7 is even. This makes the statements "a is odd" and "a is an integer and 3a+7 is even" logically equivalent. **(1 pt for stating they are logically equivalent.)**

Grading = 6 pts total allocated above.