Spring 2014 COT 3100 Multiple Choice Exam #2
VERSION A SOLUTIONS

1) ***<u>Did you bubble your PID and exam version on your scantron? (Note: You will only get credit for this question if your answer is Yes and if your answer is accurate.)</u>***

a) Yes b) No

A: You only got credit if you bubbled a AND really did bubble in your exam version and PID.

2) How many divisors does $2^3 3^5 5^2$ have?

a) 3 b) 10 c) 30 d) 72 e) None of the Above

D: Using the formula given in class, this number has (3 + 1)(5 + 1)(2 + 1) = 72 divisors.

3) What is the sum of divisors of $2^4 3^3$?

a) 31 x 40 b) 16 x 27 c) 32 x 41 d) 6^7 e) None of the Above

A: Using the formula given in class the sum of this number's divisors is (1 + 2 + 4 + 8 + 16)(1 + 3 + 9 + 27) = 31 x 40.

4) Which of the following is NOT a prime number?

a) 2 b) 17 c) 73 d) 91 e) 103

D: Since 91 = 7 x 13, it is not prime.

5) Which of the following is equivalent to 17 mod 31?

a) -45 b) -17 c) -79 d) 14 e) None of the Above

A: Since 17 – (-45) = 62 and 31 | 62, by definition of mod, -45 ≡ 17 mod 31, as desired

6) Let a, b, c and d be positive integers such that a | b and c | d. Which of the following assertions is always true?

a) (a + c) | (b + d) b) ab | cd c) ac | bd d) ad | bc e) None of the Above

C: Using the given information there are integers x and y such that b = ax, d = cy. It follows that bd = ac(xy). By definition of divisibility, this means that ac | bd. It's relatively simple to construct counter-examples that disprove the other options listed.

7) When dividing 183 by 22, we obtain a quotient q and a remainder r, as defined by the division algorithm. What is $q+r$?

a) 7 b) 8 c) 15 d) 24 e) None of the Above

C: 183 = 8 x 22 + 7, so q = 8 and r = 7 and their sum is 15.

8) What is the greatest common divisor of 28 and 182?

a) 1 b) 2 c) 7 d) 14 e) None of the Above

D: 182 = 28 x 6 + 14 and 28 = 2 x 14, so gcd(182, 28) = 14

9) What is the greatest common divisor of 24 and 66?

a) 2 b) 6 c) 12 d) 24 e) None of the Above

B: 66 = 2 x 24 + 18, 24 = 1 x 18 + 6, 18 = 3 x 6, so gcd(66, 24) = 6

10) What is the least common multiple of 24 and 66?

a) 66 b) 132 c)792 d) 1584 e) None of the Above

E: lcm(24, 66) = 66 x 24 / gcd(66, 24) = 66 x 24 / 6 = <u>264</u>, which isn't listed.

11) According to the course textbook, all ISBN-10 numbers $d_1d_2 \ldots d_{10}$ satisfy the following property: $\sum_{i=1}^{10} ix_i \equiv 0 \ (mod\ 11)$. If the first 9 digits of an ISBN-10 number are all 2, what is the last digit?

a) 2 b) 3 c) 4 d) 5 e) None of the Above

A: The sum of the first 9 terms is 2(1 + 2 + 3 + … + 9) = 90. We need x_{10} such that 90 + $10x_{10}$ is divisible by 11. The next multiple of 11 after 90 is 99, so we need $10x_{10} \equiv 9$ (mod 11). For small numbers like this, we can just plug in different values of x_{10} until we find that 10(2) ≡ 9 (mod 11) as desired. Thus, the last digit must also be 2. The more formal way to solve this is to find 10^{-1} (mod 11), which is 10, and multiply the whole equation through by that to reveal x_{10}.

12) Which of the following parts is NOT part of a proof by mathematical induction?

a) base case b) inductive hypothesis c) counter-example d) inductive step e) None of the Above

C: The three key pieces of a proof by mathematical induction are the base case, inductive hypothesis and inductive step. Thus, C is the only item listed that is NOT part of an inductive proof, typically.

13) Which of the following statements is true?

a) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + f(2n+2)$

b) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + f(2n+1) + f(2n+2)$

c) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + (\sum_{i=1}^{2n+2} f(i))$

d) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + (\sum_{i=2n}^{2n+2} f(i))$

e) None of the Above

B: We must plug in each integer from 1 to 2n + 2 into function f, according to each summation listed. In choice B, we first plug in the list of values 1, 2, …, 2n. After that we separately plug in 2n + 1 and 2n + 2. In choice A, we are forgetting to plug in 2n + 1. In choice C, we are adding most terms twice, and in choice D we are adding the term f(2n) twice and the rest of the terms once.

14) What is the difference between regular mathematical induction and strong induction?

a) The strong inductive hypothesis assumes that the given formula is true for potentially several different values instead of just one value.

b) Strong induction does not require proving the inductive step.

c) Strong induction does not require proving any base cases.

d) Strong induction requires proving two different inductive steps.

e) None of the Above

A: We assume the strong inductive hypothesis for all values of n less than or equal to some fixed k. Thus, this assumption may include more than one value. In regular induction, we just make the assumption about a single fixed value, k.

15) Consider proving that $5 \mid (n^5 – n)$ using mathematical induction for all non-negative integers. Would the proof involve algebra that required expanding a binomial of the form $(k + 1)^5$?

a) Yes　　　　b) No

A: In using induction, we'd get an expression of the form $(k + 1)^5 – (k + 1)$ in our inductive step. At this point, the most logical thing to do (and what works), is expanding out $(k + 1)^5$ and then using algebra and the inductive hypothesis to factor out a 5 from everything that's left.

16) If I want to prove that A > B, which of the following set of steps will prove it?

a) A > C > D > E > B

b) A > C < D > E > B

c) A < C < D < E < B

d) A > E > D < C > B

e) None of the Above

A – We can show that one quantity is bigger than another by showing that it's bigger than an intermediate quantity, which is bigger than another intermediate quantity, and so on. We can NOT, however, flip signs in the middle of the proof, which is done in cases B, C and D.

17) Let H_n stand for the n^{th} Harmonic number. What is H_3?

a) 1 b) 1.5 c) 2 d) 3 e) None of the Above

E: H_3 is 1 + ½ + 1/3 = 11/6. This isn't an answer choice.

18) Consider the equation $17x \equiv 14 \pmod{44}$. If we know that 17^{-1} mod 44 equals 13, what value of x satisfies the equation?

a) 1 b) 3 c) 6 d) 10 e) None of the Above

C: Multiply the equation through by 13. This yields $17(13)x \equiv 14(13) \pmod{44}$. Reducing mod 44, we get $x \equiv 6 \pmod{44}$, since $14 \times 13 = 182 \equiv 6 \pmod{44}$.

19) If we want to test whether or not 9973 is prime, what is the largest prime number we have to try to divide into it? (Note: 9973 is very close to 10000.)

a) 83 b) 97 c) 99 d) 9967 e) None of the Above

B: 97 is the largest prime less than 100 and we only need to try primes upto 100 since sqrt(9973) is in between 99 and 100. C is wrong because 99 isn't prime. D is wrong because we can stop checking after we get to 97. 83 is wrong because it's possible a larger prime divides into the value given.

20) March 14^{th} is often known as Pi day because 3.14 is an approximation to what well-known mathematical constant?

a) π b) e c) Avogadro's Number d) i e) φ