<u>**Spring 2014 COT 3100 Multiple Choice Exam #1 Version A**</u>

1) For how many combinations of values of p and q is $p \rightarrow q$ true?

a) 0 b) 1 c) 2 d) 3 e) 4

D – The simplified expression is $\bar{p} \vee q$, which is true for 3 of the 4 possible truth settings. It's false for p = true, q = false.

2) What is the inverse of $\bar{q} \rightarrow p$? (Note: Only choose an answer that applies the definition of converse to this statement directly. Do NOT, for example, choose the contrapositive of the converse.)

a) $p \rightarrow q$ b) $p \rightarrow \bar{q}$ c) $q \rightarrow p$ d) $\bar{p} \rightarrow \bar{q}$ e) None of the Above

E – The actual converse of the given expression is $q \rightarrow \bar{p}$, which isn't listed.

3) Which Logic Law is being applied in the example below?

$$(\bar{r} \wedge q) \vee (\bar{r} \wedge \bar{p}) \leftrightarrow \bar{r} \wedge (q \vee \bar{p})$$

a) Commutative b) Associative c) Distributive d) De Morgan's
e) None of the Above

C – This is written in the reverse order of the Distributive property on the formula sheet, so that's the correct answer.

4) In class we covered a Boolean expression that checked the satisfiability of a Sudoku puzzle. In particular, the variable p(i, j, n) is set to true iff the cell in row i, column j has the value n. Which constraint is checked by the following Boolean expression?

$$\bigwedge_{j=1}^{9} \bigwedge_{n=1}^{9} \bigvee_{i=1}^{9} p(i,j,n)$$

a) Each row contains every number b) Each column contains every number
c) Each 3 x 3 box contains every number d) The sum of each row is 45.
e) None of the above

B – The outer AND is iterating through each column. The two inner "loops" ensure that each number appears in column j.

5) Imagine that your goal is to disprove $\forall x \exists y P(x, y)$. If you could prove which of the following would you successfully disprove the claim?

a) $\forall y \overline{P(7, y)}$ b) $\exists y \overline{P(3, y)}$ c) $\forall y P(5, y)$ d) $\exists y P(4, y)$
e) None of the Above

A – To disprove a for all statement, you must show the existence of some x for which that statement isn't true. What choice A does is show that for x = 7, there isn't a single y for which the given statement is true. This contradicts the statement that for all x, there exists some y that makes the statement true.

6) Which of the following is equivalent to $\nexists x P(x)$?

a)$\forall x P(x)$ b) $\forall x \overline{P(x)}$ c) $\exists x P(x)$ d) $\exists x \overline{P(x)}$
e) None of the Above

B – If there isn't an x for which the statement is true, then for all x, the statement is false.

7) Given the following premises, which of the following conclusions can be drawn with confidence?

i) $p \rightarrow q$ ii) $\bar{r} \rightarrow \bar{q}$ iii) p

a) r b) $\bar{r}$ c) $\bar{q}$ d) $q \rightarrow p$ e) None of the Above

A – Use the contrapositve of (ii) to show that $q \rightarrow r$, and the Law of Syllogism and (i) to conclude that $p \rightarrow r$. Finally, invoke (iii) with Modus Ponens to prove that r must be true.

8) Consider the following statement: if a is an odd integer, then (3a + 1)(2a - 2) is divisible by 8. Which proof technique is used below to prove it?

Assume a is odd. Then there exists an integer n such that a = 2n + 1. Substitute into the given expression:

(3a + 1)(2a - 2) = (3(2n+1) + 1)(2(2n+1) - 2) = (6n + 4)(4n) = 8n(3n+2).

Since n is an integer, n(3n + 2) is as well, and the given expression is divisible by 8.

a) Proof by Contradiction b) Proof of the Contrapositive c) Proof of the Inverse
d) Direct Proof e) None of the above.

D – We start with the given information and make deductions. That is direct proof.

9) In class you saw a proof that $\sqrt{2}$ is irrational. Which proof technique was used to derive this result?

a) Proof by Contradiction b) Proof of the Contrapositive c) Proof of the Inverse
d) Direct Proof e) None of the above.

A – In the beginning of the proof we assume that $\sqrt{2}$ can be expressed as a fraction. Thus, the technique used is proof by contradiction.

10) The statement, "For all positive real values x and y, $\frac{x+y}{2} > \sqrt{xy}$", is false. Which of the following pairs of values for x and y provide a counter-example that disproves this statement?

a) x = 3, y = 5 b) x = 4, y = 4 c) x = 5, y = 3 d) x = 0, y = 0
e) None of the above

B – If you plug in 4 for both x and y, then the two quantities in question are equal. Since 4 is positive, this disproves the statement. Note that x = 0 and y = 0 also makes the quantities equal, but the statement says nothing about what occurs when you plug in values that aren't positive, so this choice DOESN'T disprove the statement as written.

11) Which of the following is true, about the following infinite sets defined in the text (and most other math books)?

a) $Z^+ \subseteq N \subseteq R \subseteq Q$ b) $N \subseteq Q \subseteq R \subseteq Z$ c) $N \subseteq Z \subseteq R \subseteq Q$
d) $N \subseteq Z^+ \subseteq R \subseteq Q$ e) None of the Above

E – None of these are true. The set of Reals (R) is the most inclusive, but doesn't appear that way in any choice. If you flip Q and R in choice A, that would be one valid answer.

12) How many elements are in the Cartesian product of {2, 4, 5, 7, 12} and {a, q, r, z}?

a) 4 b) 5 c) 9 d) 20 e) None of the Above

D – In a Cartesian Product, we pair each item in the first set with the second. This can be illustrated in a grid. So, there are 5 x 4 = 20 elements in the Cartesian product.

13) There are 23 students in a class. All of the students either play an instrument or play a sport. If exactly 15 students play a sport and exactly 18 students play an instrument, how many students play both a sport and an instrument?

a) 0 b) 5 c) 8 d) 23 e) None of the Above

E – Using the Inclusion-Exclusion Principle, we get 23 = 15 + 18 – x, where x is the number who play both. x = 10, which isn't a choice.

14) Let $f(x) = e^{3x+4}$ and $g(x) = \sin(5x - 7)$, which of the following is g(f(x))?

a) $\sin(5(3x+4) - 7)$ b) $\sin(3^{3x+4})$ c) $\sin(3^{5x-7})$ d) $\sin(5e^{3x+4} - 7)$
e) None of the Above

D – Just plug in e^{3x+4}, wherever you see x in the expression $\sin(5x - 7)$.

15) Let $f(x) = \frac{2x-5}{7}$, for a domain of all real x. What is $f^{-1}(x)$?

a) $\frac{7}{2x-5}$ b) $\frac{7}{2}x - \frac{7}{5}$ c) $\frac{7}{2}x + \frac{5}{2}$ d) $7x + \frac{5}{2}$ e) None of the Above

C – Switch x and y in the expression and solve for y. After the first step we get 7x = 2y – 5. Add 5 to both sides, then divide by 2 to get the final answer.

16) Let L_n be sequence defined as follows: $L_1 = 1$, $L_2 = 3$, $L_n = L_{n-1} + L_{n-2}$, for all integers n > 2. What is L_7?

a) 18 b) 29 c) 47 d) 76 e) None of the Above

B – $L_3 = 4$, $L_4 = 7$, $L_5 = 11$, $L_6 = 18$, $L_7 = 29$.

17) What is $\sum_{i=0}^{\infty}(\frac{3}{4})^i$?

a) 1 b) 3 c) 9 d) 16 e) None of the Above

E – This is an infinite geometric sum with first term 1 and common ratio ¾. The sum is 1/(1 – ¾) = 4.

18) If a matrix A has dimensions 2 x 6 and the matrix product AB exists, which of the following are possible dimensions of B?

a) 2 x 2 b) 3 x 2 c) 4 x 6 d) 6 x 6 e) None of the Above

D – The number of columns in the first matrix must equal the number of rows in the second matrix. The only valid choice with 6 rows is choice D.

19) A complete undirected graph is one where each pair of distinct vertices has a single edge between it. Consider the adjacency matrix of a complete undirected graph with 6 vertices. How many entries in this adjacency matrix would store a 1?

a) 30 b) 33 c) 35 d) 36 e) None of the Above

A – There are 6 entries in the whole adjacency matrix, of which the forward diagonal must be all 0s. 6 x6 – 6 = 36 – 6 = 30.

20) The non-profit organization (Red) uses its donations to help fight AIDS. What color is its logo?

a) red b) green c) blue d) purple e) yellow

RED