Spring 2014 COT 3100 Free Response Exam #1 Solutions

1) (10 pts) Construct a logical expression that has the following truth table. Your logical expression may only use the operators: $\wedge$, $\vee$, and $\overline{\ \ }$.

p	*q*	*r*	*Desired Values*
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

To prove your answer, construct a truth table with intermediate rows to prove that your expression is equivalent to the one shown above.

Your Expression: $\boldsymbol{r} \vee (\overline{\boldsymbol{p} \vee \boldsymbol{q}})$ **(Any correct expression is 5 pts)**

p	*q*	*r*	$\boldsymbol{p} \vee \boldsymbol{q}$	$\overline{\boldsymbol{p} \vee \boldsymbol{q}}$	$\boldsymbol{r} \vee (\overline{\boldsymbol{p} \vee \boldsymbol{q}})$
F	F	F	F	T	T
F	F	T	F	T	T
F	T	F	T	F	F
F	T	T	T	F	T
T	F	F	T	F	F
T	F	T	T	F	T
T	T	F	T	F	F
T	T	T	T	F	T

Grading: 5 pts for intermediate rows. There needs to be at least 1. Take 1 pt off for each incorrect entry, capping at 5.

Max points for any incorrect expression = 4 (0 for first part, 4 for second part…if it's incorrect, there must be at least one error in the chart.)

2) (10 pts) Recall that the definition of Cartesian Product of two sets A and B is as follows:

$$A \ x \ B = \{(a, b) | a \in A \land b \in B\}$$

For example, if A = {1,3} and B = {2,4,6}, then A x B = {(1, 2),(1, 4),(1, 6),(3, 2),(3, 4),(3, 6)}.

Prove or disprove the following statement for arbitrary sets A, B and C:

if $A \subseteq B$, then $AxC \subseteq BxC$.

This statement is true. (2 pts) We use direct proof to prove it.

We must show: $\boldsymbol{AxC \subseteq BxC}$. (1 pts)

This means that for an arbitrarily chosen element $\boldsymbol{(x, y) \in AxC}$, we must show that $\boldsymbol{(x, y) \in BxC}$. (2 pts)

Using the definition of Cartesian Product, and the given information, it follows that $\boldsymbol{x \in A \land y \in C}$. (2 pts)

Since $\boldsymbol{A \subseteq B}$ and $\boldsymbol{x \in A}$, by definition of subset it follows that $\boldsymbol{x \in B}$. (2 pts)

Finally, since $\boldsymbol{x \in B \land y \in C}$, by definition of Cartesian Product, it follows that $\boldsymbol{(x, y) \in BxC}$, as desired. (1 pt)

Grading: For other proof techniques, map points as needed.

3) (10 pts) Use the laws of implication and laws of logic to prove the conclusion shown from the following premises:

$(p \vee q) \rightarrow (r \vee s)$
$(r \vee s) \rightarrow t$
$\bar{t}$
$\bar{p} \rightarrow u$
$\bar{q} \rightarrow v$

$u \wedge v$

Note: You may not use all of the rows shown below.

Step	Reason
1. $(\boldsymbol{p} \vee \boldsymbol{q}) \rightarrow (\boldsymbol{r} \vee \boldsymbol{s})$	**Given**
2. $(\boldsymbol{r} \vee \boldsymbol{s}) \rightarrow \boldsymbol{t}$	**Given**
3. $(\boldsymbol{p} \vee \boldsymbol{q}) \rightarrow \boldsymbol{t}$	**Law of Syllogism (1, 2)**
4. $\bar{\boldsymbol{t}}$	**Given**
5. $\overline{\boldsymbol{p} \vee \boldsymbol{q}}$	**Modus Tollens (3, 4)**
6. $\bar{\boldsymbol{p}} \wedge \bar{\boldsymbol{q}}$	**De Morgan's Law (5)**
7. $\bar{\boldsymbol{p}}$	**Conjunctive Simplification (6)**
8. $\bar{\boldsymbol{p}} \rightarrow \boldsymbol{u}$	**Given**
9. $\boldsymbol{u}$	**Modus Ponens (7, 8)**
10. $\bar{\boldsymbol{q}}$	**Conjunctive Simplification (6)**
11. $\bar{\boldsymbol{q}} \rightarrow \boldsymbol{v}$	**Given**
12. $\boldsymbol{v}$	**Modus Ponens (10, 11)**
13. $\boldsymbol{u} \wedge \boldsymbol{v}$	**Rule of Conjunction (9, 12)**
14.	
15.	
16.	
17.	
18.	
19.	
20.	

Grading: ½ a point off per error (either wrong name or invalid step), round down, cap at 10 off.

4) (10 pts) What is the following sum equal to in terms of n: $\sum_{i=1}^{2n+1}(3i^2)$? Please leave your answer in factorized form.

$$\sum_{i=1}^{2n+1}(3i^2) = \frac{3(2n+1)(2n+2)(4n+3)}{6} = (2n+1)(n+1)(4n+3)$$

Grading: 5 pts for correctly plugging in 2n+1 into the appropriate formula, 5 pts for simplifying