r- Permutations (without repetition)

$$P(n,r) = \frac{n!}{(n-r)!}$$

counts the number of ways of selecting r elements from $\{1,2,3,\dots,n\}$ when order matters

r- Combinations (without repetition)

same as above but order does not matter

$$C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

r-Permutations with repetition

n^r

r-Combinations with repetition

apples, oranges, pears

4 pieces of fruit

4 apples

3 apples, 1 orange

3 apples, 1 pear

$\vdots$

15 ways

r-Combinations with Repetition

$| \;|$

$|* * * *|$ 0 apples 4 oranges 0 pears

$* | * | * *$ 1 apple 1 orange 2 pears

$n = 3$

$r = 4$

$C(r + (n-1), r)$

$\binom{6}{2, 4} = 15$

Permutations with Indistinguishable Objects

SUCCESS

$$C(7,3)\,C(4,2)\,C(2,1)\,C(1,1)$$

$$= \frac{7!}{3!\,4!}\,\frac{4!}{2!\,2!}\,\frac{2!}{1!\,1!}\cdot\frac{1!}{1!\,0!} =$$

$$0! = 1$$

$$= \frac{7!}{3!\,2!\,1!\,1!} = 420$$

Permutations with Indistinguishable Objects

n_1 objects of type 1

n_2 2

$\vdots$

n_k k

total number n

$$\frac{n!}{n_1!\, n_2! \cdots n_k!}$$

multinomial coefficient $= \binom{n}{n_1; n_2; \ldots; n_k}$

$$\binom{n}{k} = \binom{n}{k;\, n-k}$$

bionomial
coefficient