Intro to Discrete Structures
Lecture 15

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Second Midterm Exam

- Tuesday 03/23/10
- Relevant for exam:
  - Topics and problems from Homework 2 and 3
  - Mathematical induction and strong induction (today's class)
- Not relevant for exam:
  - Section 3.1 Algorithms, Section 3.2 Growth of Functions, Section 3.3 Complexity of Algorithms, Section 3.8 Matrices

Out of Town

- I'll be out of town from Monday 03/22 till Friday 03/26.
- The GTAs will run the second midterm exam on Tuesday 03/23.
- No class on Thursday 03/25

Fibonacci numbers

Definition 1: The Fibonacci numbers f_0, f_1, f_2, . . . are defined by

$$
\begin{aligned}
f_0 &= 0 \\
f_1 &= 1 \\
f_n &= f_{n-1} + f_{n-2} \quad \text{for } n \geq 2.
\end{aligned}
$$

$$
\begin{aligned}
f_2 &= 0+1 = 1 \\
f_3 &= 1+1 = 2 \\
f_4 &= 1+2 = 3
\end{aligned}
$$

Growth of the Fibonacci Numbers

Example 6: Show that whenever $n \geq 3$, $f_n > \alpha^{n-2}$, where $\alpha = (1+\sqrt{5})/2$.

Example 6 on page 297

Prove that $\sum_{i=0}^{n} F_i^2 = F_n F_{n+1}$.

<u>basis case</u>

$n = 0$ $\quad \sum_{i=0}^{0} F_i^2 = F_0^2 = 0^2 = 0$ $\quad$ LHS

$F_0 \cdot F_1 = 0 \cdot 1 = 0$ $\quad$ RHS

Basis case is true because LHS = RHS.

induction hypothesis

$$\sum_{i=0}^{n} F_i^2 = F_n F_{n+1}$$ holds for some n

inductive step

We have to show that

$$\sum_{i=0}^{n+1} F_i^2 = F_{n+1} F_{n+2}$$ using IH.

$$\sum_{i=0}^{n+1} F_i^2 = \left(\sum_{i=0}^{n} F_i^2\right) + F_{n+1}^2 \quad \text{I.H.}$$

$$= F_n F_{n+1} + F_{n+1}^2$$

$$= F_{n+1}(F_n + F_{n+1}) \quad \text{due to the def. of the Fib. numbers}$$

$$= F_{n+1} F_{n+2}$$

Let A and B be finite sets.

$|A|$ the cardinality of A = # of elements in A

$f: A \longrightarrow B$ injective function

$$|A| \leq |B|$$

$f: A \longrightarrow B$ surjective

$$|A| \geq |B|$$

$f : A \to B$ bijective

$|A| = |B|$

$f : A \to B$ $\qquad g : B \to C$

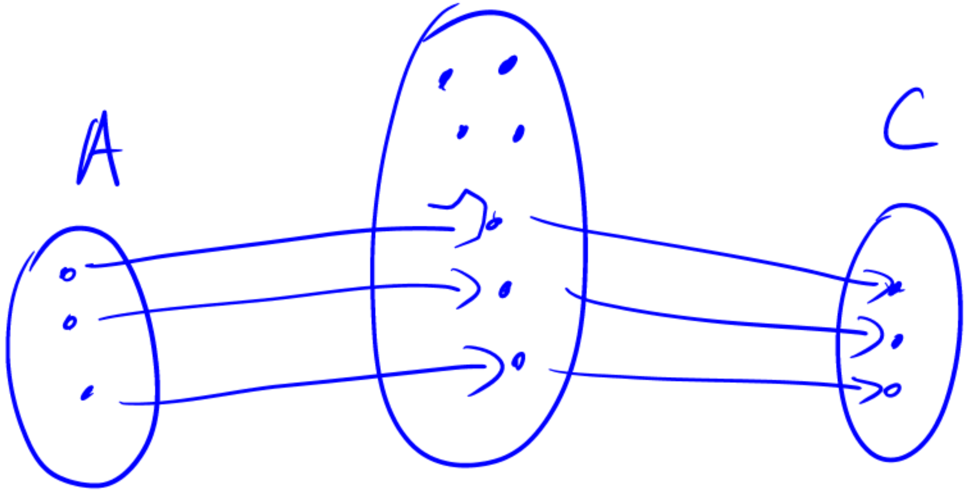
$g \circ f : A \longrightarrow C$

f not injective $\Rightarrow$ $g \circ f$ not injective

$f : A \to B$ not surjective

$g : \cancel{A}\, B \to C$

Can we <u>always</u> conclude that $g \circ f$ is not$_B$ surjective? NO



counter example

Problem: Prove that the sum of the squares of 6 consequtive non-negative integers leaves the remainde 7 when dividing by 12.

Prove that $\sum_{i=0}^{5}(n+i)^2 \equiv 7 \pmod{12}$ holds for all n.

$n = 4$ $\qquad 4^2+5^2+6^2+7^2+8^2+9^2$

$$S(n) := \sum_{i=0}^{5} (n+i)^2$$

We have to prove that

$$\forall n \geq 0 \quad S(n) \equiv 7 \pmod{12}$$

<u>basis case:</u> $S(0) \equiv 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$$\equiv \underbrace{0 + 1 + 4 + 9}_{2} + \underbrace{16}_{4} + \underbrace{25}_{1}$$

$$\equiv 2 \quad\quad 4 \quad\quad 1$$

$$\equiv 7 \pmod{12} \quad \checkmark$$

induction hypothesis:

We assume that $S(n) \equiv 7 \pmod{12}$ ⊛

inductive step:

We have to show that $S(n+1) \equiv 7 \pmod{12}$ using ⊛ and laws of arithmetics.

$$S(n+1) = S(n) + (n+6)^2 - n^2$$

$$S(n) = \sum_{i=0}^{5} (n+i)^2$$

$$S(n+1) = \sum_{i=0}^{5} (n+1+i)^2$$

$$= \sum_{i=1}^{6} (n+i)^2$$

$$\Longrightarrow \quad S(n+1) = S(n) + (n+6)^2 - n^2$$

$$S(n+1) = S(n) + (n+6)^2 - n^2$$

$$= S(n) + n^2 + 12n + 36 - n^2$$

$$= S(n) + (12n) + 36$$

$$\Rightarrow S(n+1) \equiv S(n) + 0 + 0 \pmod{12}$$

$$\equiv 7 \pmod{12}$$