Intro to Discrete Structures Lecture 14

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Mathematical Induction

Mathematical induction is based on the rule of inference

1.
$$P(1)$$

2. $\forall k (P(k) \rightarrow P(k+1))$
3. $\therefore \forall n P(n)$

which is true for the domain of natural numbers \mathbb{N} .

Climbing an Infinite Ladder

- 1. We can reach the first rung of the ladder.
- 2. If we can reach a particular rung of the ladder, then we can reach the next rung of the ladder.
- 3. Therefore, we can reach any rung of the ladder.

Principle Mathematical Induction

To prove that P(n) is true for all natural numbers n, where P(n) is a propositional function, we complete two steps:

- Basis step: We verify that P(1) is true.
- We show that the conditional statement

$$P(k) \rightarrow P(k+1)$$

is true for all natural numbers k.

Warning

- In a proof of mathematical induction is is **not** assumed that P(k) is true for all k.
- It is only shown that **if it is assumed** that P(k) is true, then P(k+1) is also true.
- Thus, a proof of mathematical induction is not a case of begging the question, or circular reasoning.

Example: Show that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Example 2: Conjecture a formula for the sum of the first n positive integers. Then prove your conjecture using mathematical induction.

Find a formula for

$$\sum_{j=1}^{n} (-1)^j j$$

The Number of Subsets of a Finite Set

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DeMorgan for Intersection

Example 10: Prove

$$\bigcap_{j=1}^{n} A_j = \bigcup_{j=1}^{n} \overline{A_j}$$

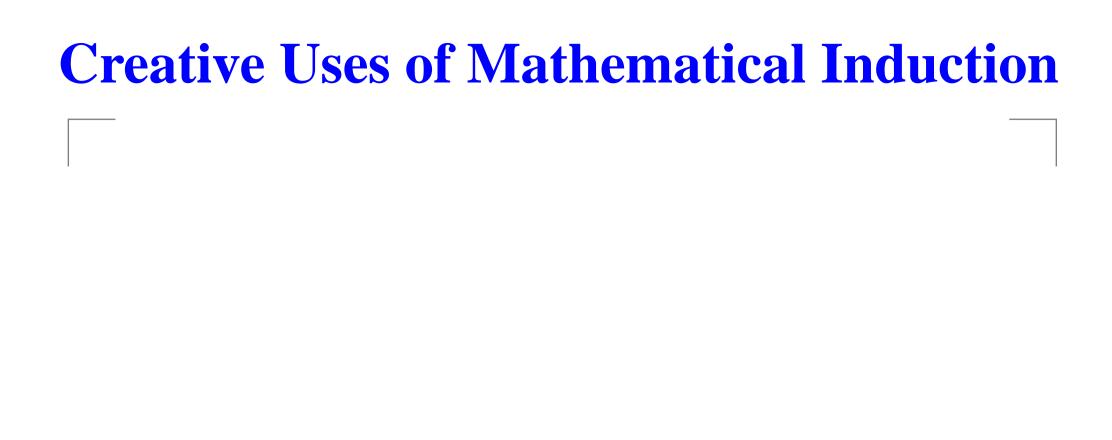
whenever A_1, A_2, \ldots, A_n are subset of a universal U and $n \geq 2$.

DeMorgan for Intersection

DeMorgan for Intersection

Creative Uses of Mathematical Induction

Example: Show that every $2^n \times 2^n$ checkerboard with one square removed can be tiled using L-shaped triominoes.



Strong Induction

Strong induction is based on the rule of inference

1.
$$P(1)$$

2. $\forall k (\wedge_{j=1}^k P(j) \rightarrow P(k+1))$
3. $\therefore \forall n P(n)$

which is true for the domain of natural numbers \mathbb{N} .

Strong Induction

To prove that P(n) is true for all natural numbers n, where P(n) is a propositional function, we complete two steps:

- **Description** Basis step: We verify that P(1) is true.
- We show that the conditional statement

$$\bigwedge_{j=1}^{k} P(j) \to P(k+1)$$

is true for all natural numbers k.

Existence of Prime Factorization

Example: Show that if n is an integer greater than 1, then n can be written as the product of primes.

Existence of Prime Factorization

Existence of Prime Factorization

Example 2: Show that if n is an integer greater than 1, then n can be written as the product of primes.

Winning Strategy

Example 3:

Winning Strategy

Winning Strategy