Intro to Discrete Structures
Lecture 10

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Membership Table

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$

Generalized Union

Definition 6: The **union** of a collection of sets is the set that contains those element that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Generalized Intersection

Definition 6: The **intersection** of a collection of sets is the set that contains those element that are members of all sets in the collection.

We use the notation

$$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Functions

Definition 1: Let A and B be nonempty sets.

A **function** f from A to B is an assignment of exactly one element of B to each element A.

We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A.

We write $f\ : A \rightarrow B$ if f is a function from A to B.

Functions

Domain / Codomain

Definition 2: If $f\ A \to B$, we say that A is the **domain** and B the **codomain** of F.

If $f(a) = b$, we say that b is the **image** of a and b is the preimage of b.

The **range** of f is the set of all images of elements of A.

If $f\ A \to B$, we also say that f **maps** from A to B.

Image

Definition: Let $S \subseteq A$ and $f : A \to B$. The **image** of S under the function f is the set

$$f(S) = \{t \in B \mid \exists s \in S\,(t = f(s))\}\,.$$

Injective Functions

Definition 5: The function $f : A \to B$ is called **injective** or **one-to-one** iff

$$\forall a \in A \, \forall b \in A \, (f(a) = f(b) \to a = b) \, .$$

Surjective Functions

Definition 7: The function $f : A \rightarrow B$ is called **surjective** or **onto** iff

$$\forall b \in B \, \exists a \in A \, (f(a) = b)) \, .$$

Bijective Function

Definition 8: The function $f : A \longrightarrow B$ is called **bijective** if it is both surjective and injective.

Examples

Inverse Function

Definition 9: Let $f : A \to B$ be a bijective function.

Then, the **inverse function** f^{-1} is the function from B to A that assigns to $b \in B$ the unique element $a \in A$ such that $f(a) = b$.

Composition of Functions

Let $g : A \longrightarrow B$ and $f : B \longrightarrow C$. The **composition** of the functions f and g, denoted by $(f \circ g)$, is defined by

$$(f \circ g)(a) = f(g(a))$$

Composition of f and f^{-1}

Graphs of Functions

Let $f : A \longrightarrow B$. The **graph of the function** f is the subset of $A \times B$ defined by

$$\{(a, b) \in A \times B \mid f(a) = b\}.$$

Examples

The graph of $f(n) = 2n + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$.

Examples

The graph of $f(x) = x^2$ from $\mathbb{Z}$ to $\mathbb{Z}$.
The graph of $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$.

Floor and Ceiling Functions

Definition 12: The **floor function** assigns to the real number x the largest integer that is less or equal to x.

The value of the floor function is denoted by $\lfloor x \rfloor$.

The **ceiling function** assigns to the real number x the smallest integer that is greater or equal to x.

The value of the ceiling function is denoted by $\lceil x \rceil$.

Functions Graphs of Floor and Ceiling

Factorial Function

The **factorial function** $f : \mathbb{N} \to \mathbb{Z}^+$ is denoted by $f(n) = n!$.

The value of $f(n) = n!$ is the product of the first n positive integers, so

$$n! = 1 \cdot 2 \cdots (n-1) \cdot n$$

and $f(0) = 1$.

(Strictly) Increasing / Decreasing

Definition 6: Let $f : A \to B$ with $A, B \subseteq \mathbb{R}$.

The function f is called

- **increasing** if

$$f(x) \leq f(y)$$

- **strictly increasing** if

$$f(x) < f(y)$$

whenever $x < y$.

Addition / Multiplication of Functions

Definition 3: Let $f_1, f_2 : A \to \mathbb{R}$.

Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to $\mathbb{R}$ defined by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x)\,, \tag{1}$$

$$(f_1 f_2)(x) = f_1(x) f_2(x)\,. \tag{2}$$

Sequences and Summations

Sequences

Summation