Intro to Discrete Structures
Lecture 9

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Sets

Definition 1: A **set** is an unordered collection of objects.

Definition 2: The objects of a set are called the **elements**, or **members**, of the set.

A set is said to **contain** its elements.

We use the notation

$$x \in A$$

to indicate that x is an element of the set A.

Example of Sets

Sets

Definition 3: Two sets are **equal** if and only if they have the same elements. That is, if A and B are sets, then A and B are equal iff

$$\forall x(x \in A \leftrightarrow x \in B) .$$

We write $A = B$ if A and B are equal sets.

Subset

Definition 4: The set A is said to be a **subset** of B iff every element of A is also an element of B.

We use

$$A \subseteq B$$

to indicate that A is a subset of B.

We see that $A \subseteq B$ iff the quantification

$$\forall x (x \in A \rightarrow x \in B)$$

is true.

Venn Diagrams

Sets

Theorem 1: For every set S, we have

- $\emptyset \subseteq S$ and
- $S \subseteq S$.

Proper Subset

When we wish to emphasize that a set A is a subset of the set B but $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B.

$$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$

Infinite Sets

Definition 6: A set is said to be **infinite** if it is not finite.

The Power Set

Definition 7: Given a set S, the **power set** is the set of all subsets of S. The power set of S is denoted by $P(S)$.

Example 13: What is the power set of $\{0, 1, 3\}$?

The power set $P(\{0, 1, 3\})$ is the set of all subset of $\{0, 1, 3\}$. Hence,

Cartesian Products

Definition 8: The **ordered n-tuple** $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its nth element.

Cartesian Products

Definition 9: Let A and B be sets. The **Cartesian product** of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\} .$$

More generally, let $A_1, A_2, \ldots, A_n$ be sets. The Cartesian product of A_1, A_2, . . . , A_n is the set

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A \text{ for } i = 1, 2, \ldots, n\} .$$

Union

Definition 1: The **union** of the sets A and B is the set

$$A \cup B = \{x \,|\, x \in A \vee x \in B\}\,.$$

Intersection

Definition 1: The **intersection** of the sets A and B is the set

$$A \cap B = \{x \,|\, x \in A \land x \in B\}\,.$$

Venn Diagrams

Disjoint Sets

Definition 3: Two set A and B are called **disjoint** iff their intersection is empty, that is,

$$A \cap B = \emptyset.$$

Difference

Definition 4: The **difference** of the sets A and B is the set

$$A - B = \{x \mid x \in A \wedge x \notin B\}\,.$$

The difference of A and B is also called the **complement of** B **respect to** A.

Definition 5: Let U be the universal set. The **complement** of A is the set

$$\bar{A} = U - A\,.$$

We have

$$\bar{A} = \{x \mid x \notin A\}\,.$$

Set Identities I

Set Identities II

Set Identities III

Example

Prove De Morgan law $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

$$
\begin{aligned}
\overline{A \cap B} &= \\
&= \\
&= \\
&= \\
&= \\
&= \\
&= \\
&=
\end{aligned}
$$

Membership Table

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$

Generalized Union

Definition 6: The **union** of a collection of sets is the set that contains those element that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Generalized Intersection

Definition 6: The **intersection** of a collection of sets is the set that contains those element that are members of all sets in the collection.

We use the notation

$$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$$

to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

Functions