Intro to Discrete Structures Lecture 9

Pawel M. Wocjan

School of Electrical Engineering and Computer Science

University of Central Florida

wocjan@eecs.ucf.edu

Sets

$\{1,2\} = \{2,1\}$

Definition 1: A **set** is an unordered collection of objects.

Definition 2: The objects of a set are called the **elements**, or **members**, of the set.

A set is said to **contain** its elements.

We use the notation

$$x \in A$$

to indicate that x is an element of the set A.

Example of Sets

$\mathbb{N} = \{1, 2, 3, \ldots\}$

The set of odd numbers

$O = \{1, \cancel{2}, 3, 5, 7, \ldots\}$

$= \{x \mid x \in \mathbb{N} \land x \text{ is odd}\}$

$S = \{1, 4, 9, 16, 25, \ldots\}$

$= \{x^2 \mid x \in \mathbb{N}\}$

$A = \{a, b, c, \ldots, z\}$

Sets

Definition 3: Two sets are **equal** if and only if they have the same elements. That is, if A and B are sets, then A and B are equal iff

isElementOfA

$$\forall x(\boxed{x \in A} \leftrightarrow \boxed{x \in B})\ .$$

$3 \in A$ False

$6 \in A$ True

We write $A = B$ if A and B are equal sets.

$A = \{2, 4, 6\}$ $C = \{2, 4\}$

$B = \{6, 2, 4\}$

Subset

Definition 4: The set A is said to be a **subset** of B iff every element of A is also an element of B.

We use

$$A \subseteq B$$

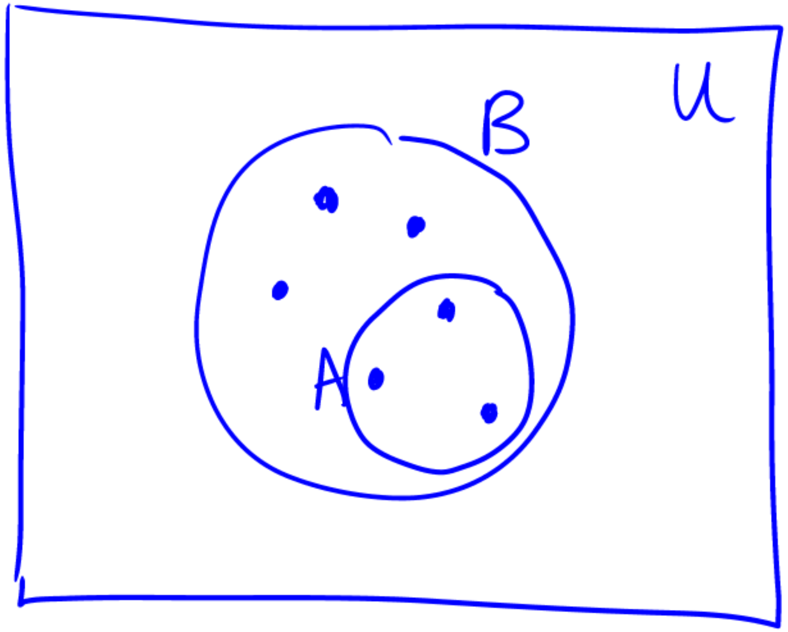
to indicate that A is a subset of B.

We see that $A \subseteq B$ iff the quantification

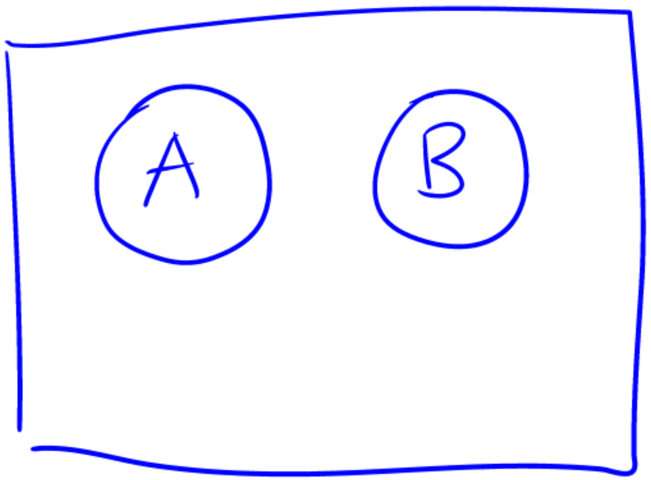
$$\forall x(x \in A \rightarrow x \in B)$$

is true.

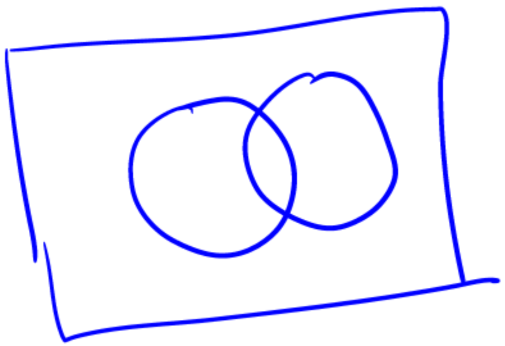
Venn Diagrams



$A \subseteq B$



$A \nsubseteq B$



Sets

Theorem 1: For every set S, we have

(1) • $\emptyset \subseteq S$ and
empty set

(2) • $S \subseteq S$.

(1) $\forall x \, (\underbrace{x \in \emptyset}_{F} \rightarrow x \in S)$

F → F
F → T

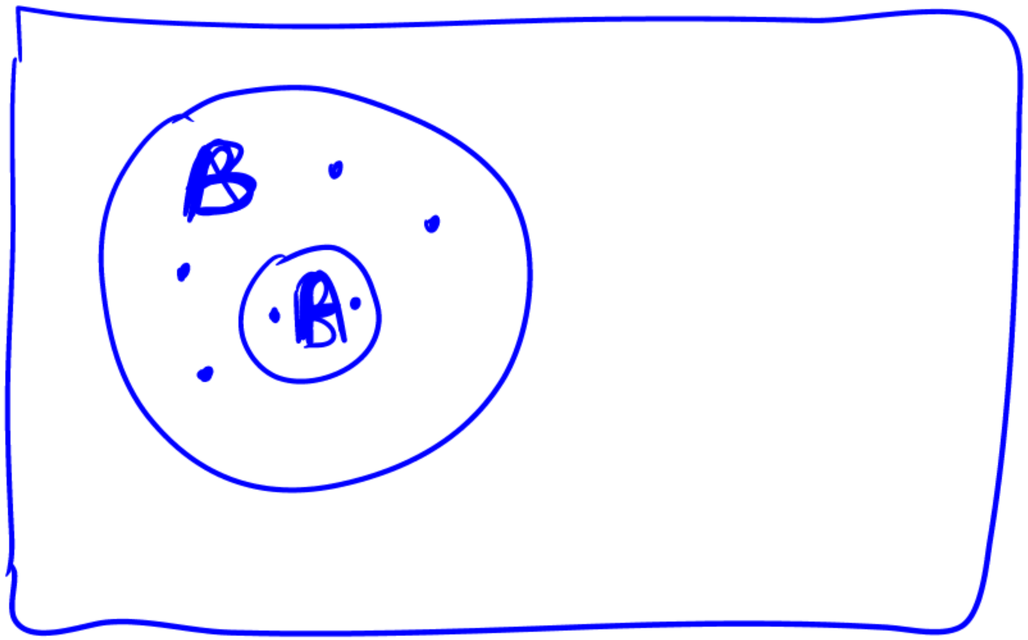
(2) $\forall x \, (x \in S \rightarrow x \in S)$

F → F
T → T

Proper Subset

When we wish to emphasize that a set A is a subset of the set B but $A \neq B$, we write $A \subset B$ and say that A is a **proper subset** of B.

$$\forall x(x \in A \rightarrow x \in B) \wedge \exists x(x \in B \wedge x \notin A)$$



$S \subseteq S$ TRUE

$S \subset S$ FALSE

Infinite Sets

Definition 6: A set is said to be **infinite** if it is not finite.

The Power Set

Definition 7: Given a set S, the **power set** is the set of all subsets of S. The power set of S is denoted by $P(S)$.

Example 13: What is the power set of $\{0, 1, 3\}$?

The power set $P(\{0, 1, 3\})$ is the set of all subset of $\{0, 1, 3\}$. Hence,

$$\{\phi, \{0\}, \{1\}, \{3\}, \{0,1\}, \{0,3\}, \{1,3\}, \{0,1,3\}\}$$

Cartesian Products

Definition 8: The **ordered n-tuple** $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, . . . , and a_n as its nth element.

Cartesian Products

Definition 9: Let A and B be sets. The **Cartesian product** of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \,|\, a \in A \wedge b \in B\}\,.$$

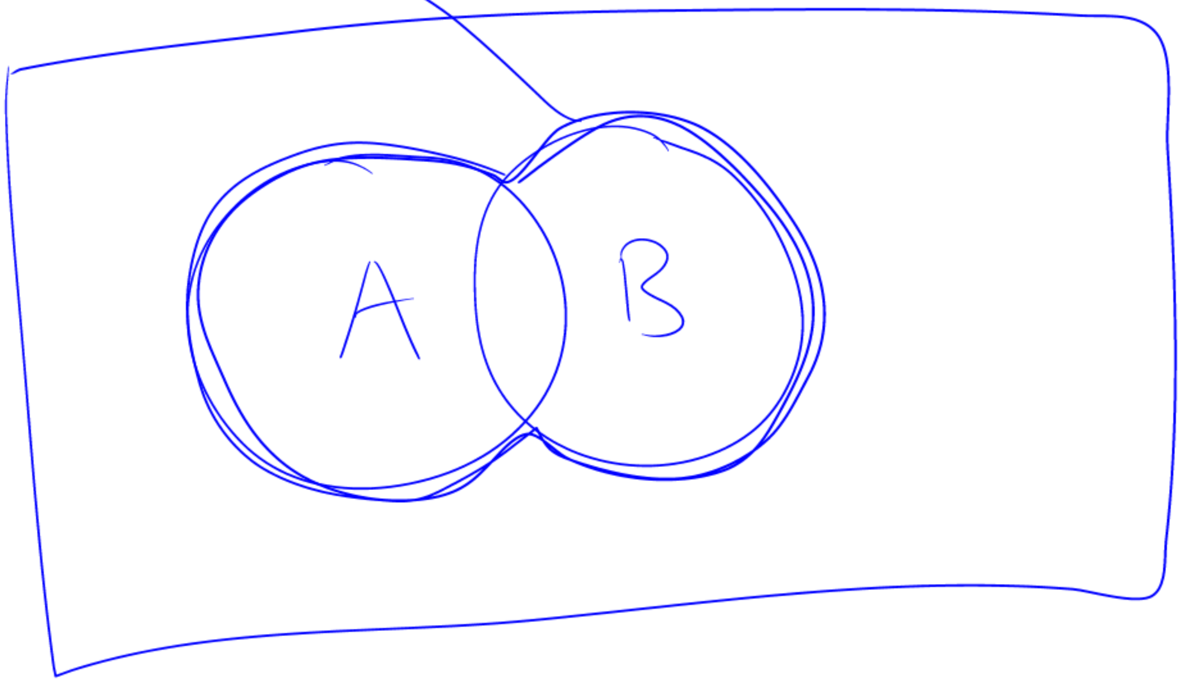
More generally, let $A_1, A_2, \ldots, A_n$ be sets. The Cartesian product of A_1, A_2, ..., A_n is the set

$$A_1 \times A_2 \times \ldots \times A_n = \{(a_1, a_2, \ldots, a_n) \,|\, a_i \in A \text{ for } i = 1, 2, \ldots, n\}\,.$$

Union

Definition 1: The **union** of the sets A and B is the set

$$A \cup B = \{x \mid x \in A \vee x \in B\}\,.$$



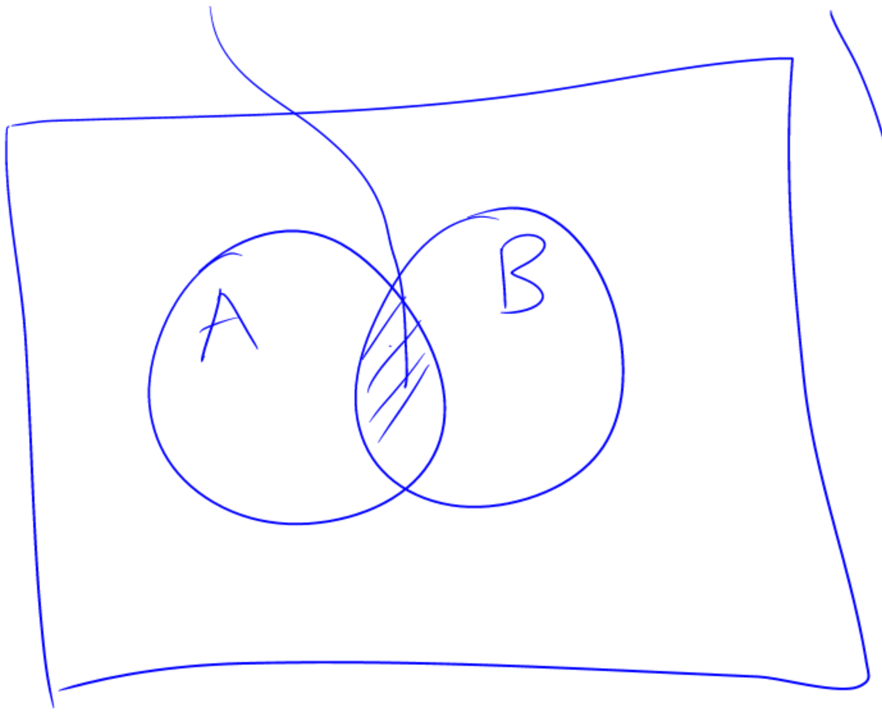
Intersection

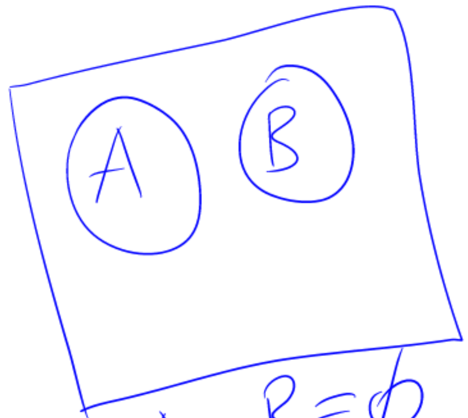
$A \subseteq B \Longrightarrow$

$A \cap B = A$

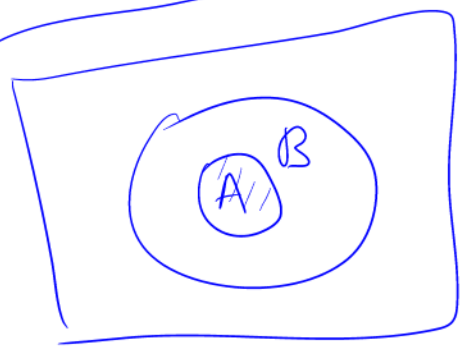
Definition 1: The **intersection** of the sets A and B is the set

$$A \cap B = \{x \mid x \in A \wedge x \in B\}\,.$$



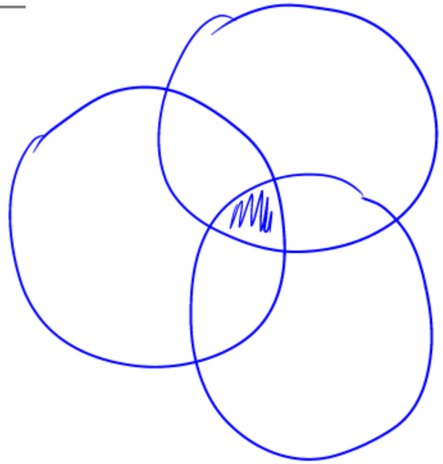


$A \cap B = \phi$

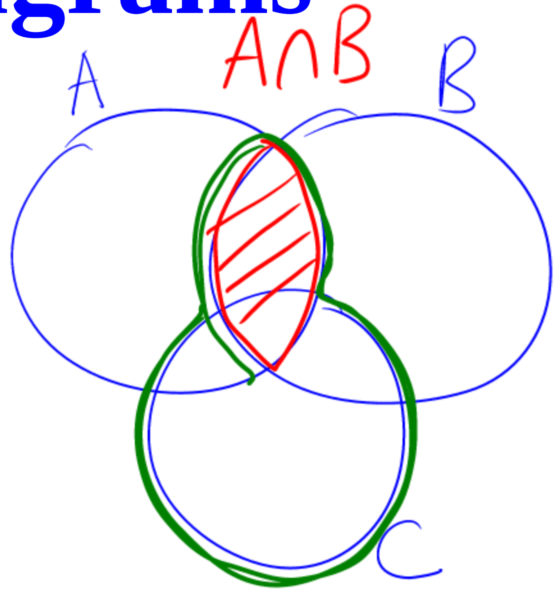


$A \cap B = A$

Venn Diagrams



$A \cap B \cap C$



$\boxed{(A \cap B) \cup C}$

Disjoint Sets

Definition 3: Two sets A and B are called **disjoint** iff their intersection is empty, that is,

$$A \cap B = \emptyset.$$

Difference

Definition 4: The **difference** of the sets A and B is the set

$$A - B = \{x \mid x \in A \land x \notin B\}.$$

The difference of A and B is also called the **complement of B respect to A**.

with

Definition 5: Let U be the universal set. The **complement** of A is the set

$$\bar{A} = U - A.$$

We have

$$\bar{A} = \{x \mid x \notin A\}.$$

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Set Identities I

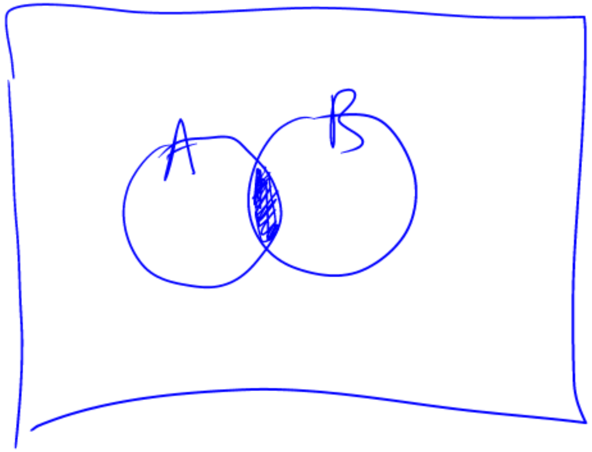
$A \cup \emptyset = A$ $A \cap U = A$	Identity Laws	$p \vee F \equiv p$ $p \wedge T \equiv p$
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination Laws	
$A \cup A = A$ $A \cap A = A$	Idempotent Laws	

Set Identities II

Set Identities III

Example

Prove De Morgan law $\overline{A \cap B} = \bar{A} \cup \bar{B}$.



$$\overline{A \cap B} = \{x \mid x \notin A \cap B\} \quad \text{def. of complement}$$

$$= \{x \mid \neg(x \in A \cap B)\} \quad \text{def. of } \notin$$

$$= \{x \mid \neg(x \in A \wedge x \in B)\}$$

$$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\}$$

$$= \{x \mid x \notin A \vee x \notin B\}$$

$$= \{x \mid x \in \bar{A} \vee x \in \bar{B}\}$$

$$= \bar{A} \cup \bar{B}$$

Membership Table

A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$

Generalized Union

Definition 6: The **union** of a collection of sets is the set that contains those element that are members of at least one set in the collection.

We use the notation

$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

to denote the union of the sets $A_1, A_2, \ldots, A_n$.

Generalized Intersection

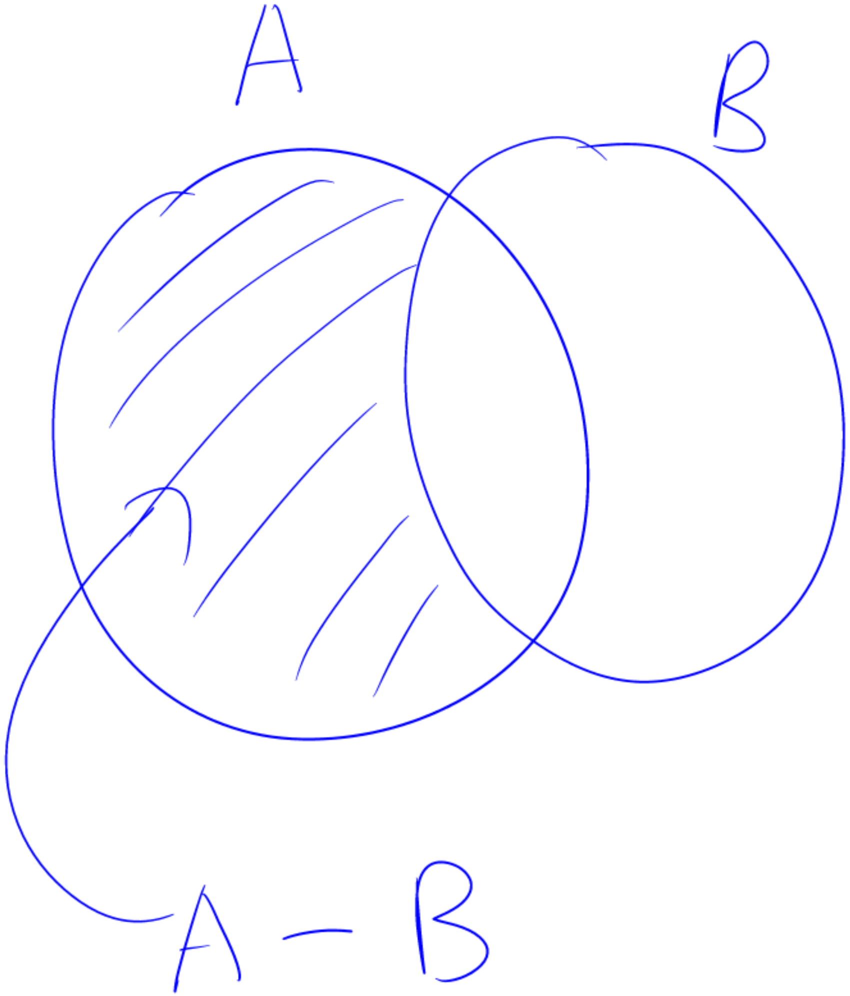
Definition 6: The **intersection** of a collection of sets is the set that contains those element that are members of all sets in the collection.

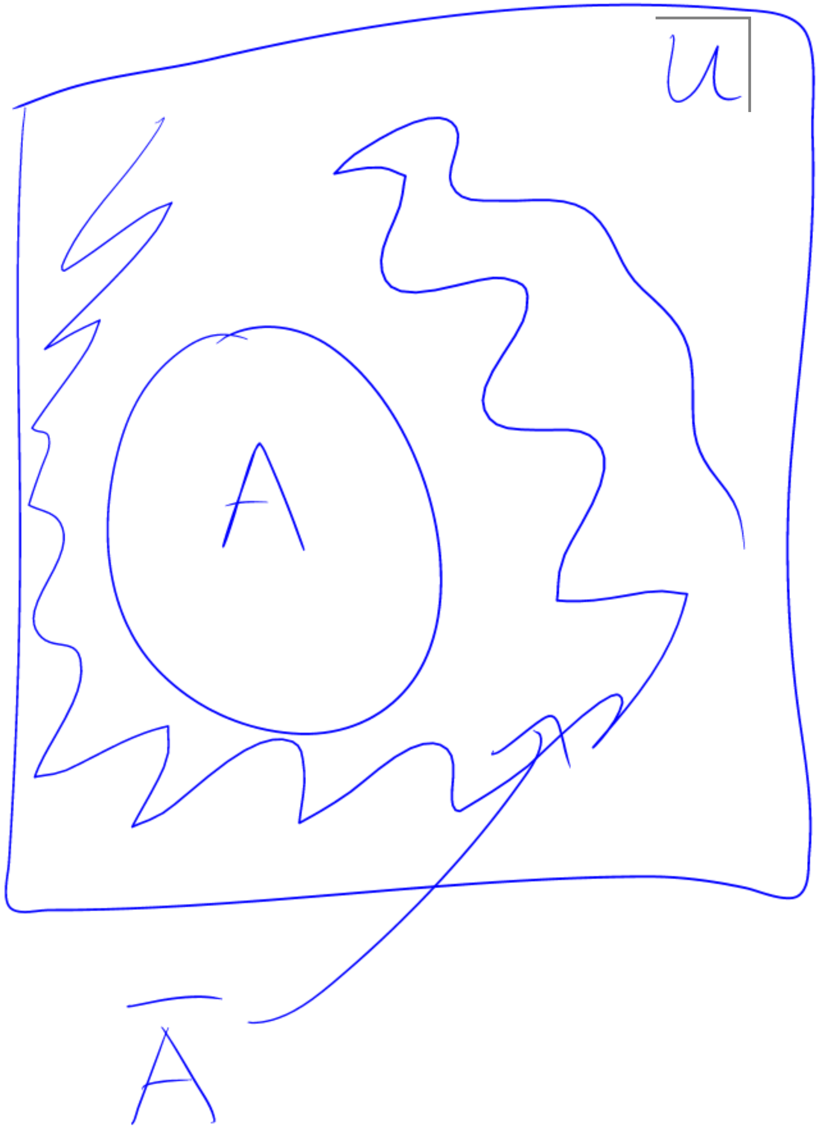
We use the notation

$$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$$

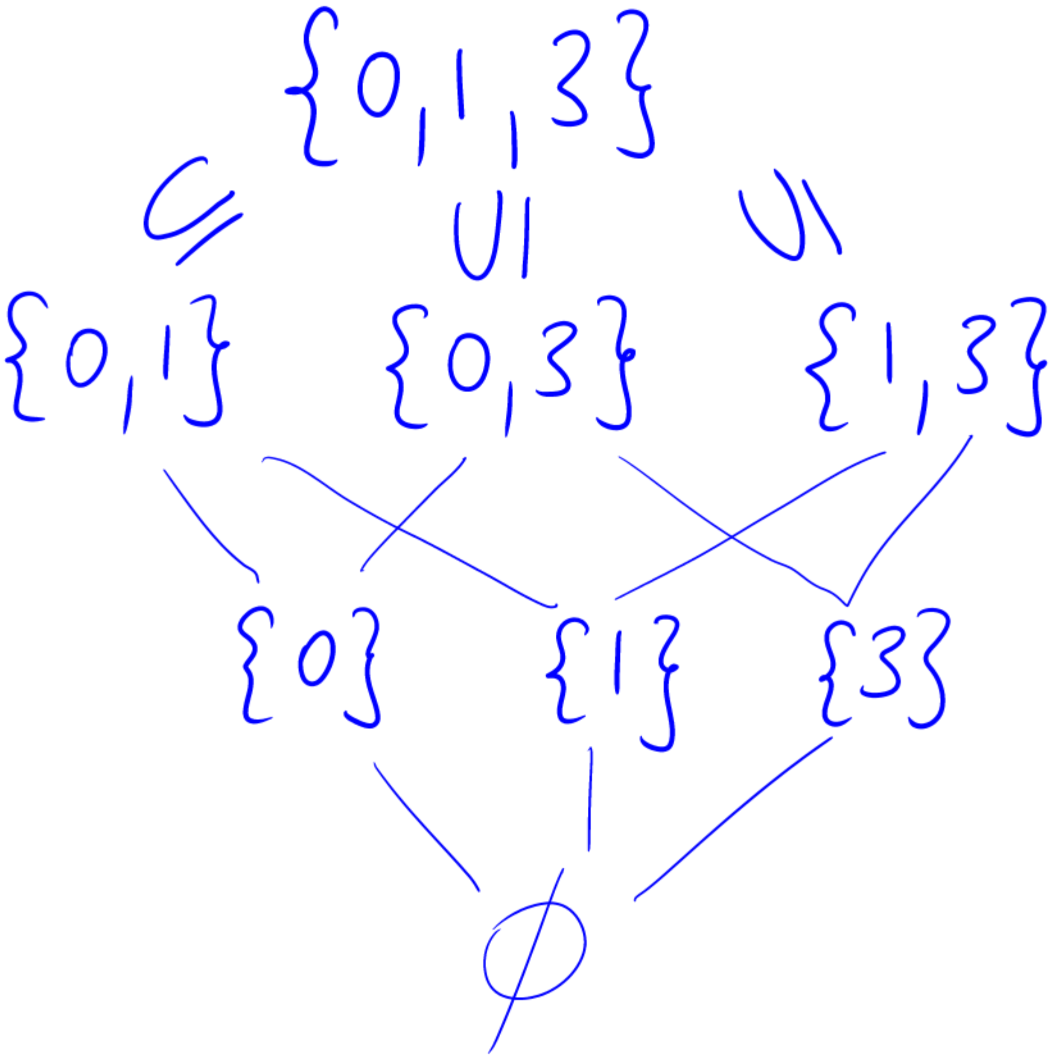
to denote the intersection of the sets $A_1, A_2, \ldots, A_n$.

~~Functions~~





partial order



total order

$$5 \geq 4 \geq 3 \geq 2 \geq 1 \geq 0$$