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Intro to Discrete Structures
Lecture 7

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Proof Strategy

- We have seen two important methods for proving theorems of the form $\forall x(P(x) \to Q(x))$.

  These two methods are

  - the direct proof and
  - the indirect proof

  methods.

- It takes some practice (solving homework problems) to learn to recognize quickly the correct approach.
- Try first the direct approach. If it does not work then try the indirect approach.

Direct or Indirect Proof?

Definition: The real number r is **rational** if there exist integers p and q with $q \neq 0$ such that $r = p/q$.

Example 7: Prove that the sum of two rational numbers is rational.

Direct or Indirect Proof?

Example 8: Prove that if n is an integer and n^2 is odd, then n is odd.

Proof by Contradition I

- Suppose we want to prove that p is true.
- Furthermore, suppose that we can find a contradition q such that

$$\neg p \longrightarrow q$$

  is true.
- Because q is false, but $\neg p \rightarrow$ is true, we can conclude that $\neg p$ is false, which means that p is true.

Proof by Contradition II

- The statement

$$(r \wedge \neg r)$$

is a contradition whenever r is a proposition.

- Therefore, we can prove that p is true if we can show that

$$\neg p \rightarrow (r \wedge \neg r)$$

is true for some proposition r.

Proof by Contradition III

Example 9: Show that at least four of any 22 days must fall on the same day of the week.

Proof by Contradition IV

- p : At least four of 22 days chosen fall on the same day of the week.
- Suppose $\neg p$ is true.
- This means that at most three days of the 22 days fall on the same day.
- This implies that at most 21 days could have been chosen.
- This contradicts the hypothesis that we have 22 days under consideration.
- r : 22 days are chosen.

$$\neg p \rightarrow (r \wedge \neg r)$$

$\sqrt{2}$ is irrational

$\sqrt{2}$ is irrational

Reading Assignment

- Examples 11 – 14
- Subsection “Mistakes in Proofs”

Exhaustive Proof

Prove that $(n+1)^2 \geq 3^n$ if n is a postive integer with $n \leq 4$.

Proof by Cases

Based on the logical equivalence

$$(p_1 \vee p_2 \vee \ldots \vee p_n) \rightarrow q \equiv (p_1 \rightarrow q) \wedge (p_2 \rightarrow q) \wedge \ldots \wedge (p_n \rightarrow q)$$

page 24 Table 6 ⎫
25 7&8 ⎭ copy attached to exam

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expected to know

$$\neg \forall x \, P(x) \equiv \exists x \neg P(x)$$

$$\neg \forall x \, (P(x) \wedge Q(x)) \equiv \exists x \neg (P(x) \wedge Q(x))$$

$$\equiv \exists x (\neg P(x) \vee \neg Q(x))$$

$$\forall x\ (x \neq 0 \rightarrow \exists y\,(xy = 1))$$
$$\equiv \forall x\ (\neg(x \neq 0) \vee \exists y\,(xy = 1))$$
$$\equiv \forall x\ (x = 0 \vee \exists y\,(xy = 1))$$

universal generalization

$P(a)$ is true for an arbitrary a from the domain

$\therefore \forall x\, P(x)$

$$\forall x \underbrace{\left(x = 0 \lor \exists y \, (xy = 1) \right)}_{P(x)}$$

- $a = 0$

  $P(0)$ $\qquad \underbrace{0 = 0}_{T} \lor \exists y \, (0 \cdot y = 1) \equiv T$

- $a \neq 0$ $\qquad \underbrace{a = 0}_{F} \lor \exists y \, (a \cdot y = 1)$

  $\equiv \exists y \, (a \cdot y = 1) \qquad y = \frac{1}{a}$

p. 28 f)

$$\exists x \forall y \, (y \neq 0 \rightarrow xy = 1)$$

$x \neq 0$

$y_1 \neq y_2$ $\quad y_1 \neq 0, \; y_2 \neq 0$

$$x y_1 = 1 = x y_2$$

$$\Rightarrow x(y_1 - y_2) = 0 \qquad \Big| \cdot \frac{1}{x}$$

$$y_1 - y_2 = 0$$

$$\Rightarrow y_1 = y_2$$

$$\forall x\,(x=0 \lor \exists y\,(xy=1))$$

Observation 1

$$x=0 \equiv \exists y\,(x=0)$$

$$\forall x\,(\underbrace{\exists y\,(x=0)}_{P(y)} \lor \exists y\,\underbrace{(xy=1)}_{Q(y)})$$

Observation 2

$$\exists x\,(P(x) \lor Q(x)) \equiv \exists x\,P(x) \lor \exists x\,Q(x)$$

$$\forall x\,\exists y\,(x=0 \lor xy=1)$$

Observation 2

$$\exists y\,(P(y) \vee Q(y)) \equiv$$
$$\exists y\,P(y) \vee \exists y\,Q(y)$$

$$\exists y\,(P(y) \wedge Q(y)) \not\equiv$$
$$\exists y\,P(y) \wedge \exists y\,Q(y) \quad ?$$

domain $\mathbb{R} \setminus \{0\}$

(a) $\forall x \exists y \; P(x,y)$

$P(x,y): x \cdot y = 1$

(b) $\exists x \forall y \; P(x,y)$

$\neg \exists x \forall y \; P(x,y)$

$\equiv \forall x \exists y \neg P(x,y)$