Intro to Discrete Structures Lecture 1

Pawel M. Wocjan

School of Electrical Engineering and Computer Science

University of Central Florida

`wocjan@eecs.ucf.edu`

1. The Foundations: Logic and Proof

- 1.1. Propositional Logic
- 1.2. Propositional Equivalances
- 1.3. Predicates and Quantifiers
- 1.4. Nested Quantifiers
- 1.5. Rules of Inference
- 1.6. Introduction to Proofs
- 1.7. Proof Methods and Strategy

1.1. Propositional Logic

The rules of logic

- give precise meaning to mathematical statements and
- are used to distinguish between valid and invalid mathematical arguments.

A major goal is to teach you how to understand and construct correct mathematical arguments.

1.1. Propositional Logic

In addition to its importance in understanding mathematical reasoning, logic has numerous applications in computer science.

For instance, these rules are used in

- the design of computer circuits,
- the construction of computer programs, and
- the verification of the correctness of programs.

Propositions

We begin with the basic building blocks of logic – propositions.

A **proposition** is a declarative sentence (that is, a sentence that declares a fact) that is either true of false, but not both.

Example 1: All the following declarative sentences are propositions.

1. Washington D.C. is the capital of the Unites States of America. True
2. Toronto is the capital of Canada. True
3. 1+1=2. True
4. 2+2=3. False

Propositions

Some sentences that are not propositions are given below.

Example 2: Consider the following sentences.

1. What time is it?
2. Read this carefully.
3. $x + 1 = 2$.
4. $x + y = z$.

Sentences 1 and 2 are not propositions because they are not declarative sentences.

Sentences 3 and 4 are not propositions because they are neither true nor false. Note that we can turn them into propositions if we assign values to the variables x and y.

Notation in Propositional Calculus

The area of logic that deals with propositions is called **propositional calculus** or **propositional logic**.

We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent propositions, just as letters are used to denote numerical variables.

The conventional letters used for propositional variables are $p, q, r, s, \ldots$.

The **truth value** of a proposition is true, denoted by T, if it is a true proposition and false, denoted by F, if it is a false proposition.

Compound Propositions

We now turn our attention to methods for producing new propositions from those that we already have.

New propositions, called **compound propostions**, are formed from existing propositions using **logical operators**.

Negation

Definition 1: Let p be a proposition. The **negation** of p, denoted by $\neg p$ (also denoted by $\bar{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of $\neg p$, is the opposite of the truth value of p.

The truth table for the negation of a proposition:

p	$\neg p$
T	F
F	T

Connectives

The negation of a proposition can also be considered the result of the operation of the **negation** operator on a proposition.

The negation operator constructs a new proposition from a *single* existing proposition.

We will now introduce the logical operators that are used to form new propositions from *two* or *more* existing propositions.

These logical operators are also called **connectives**.

Conjuction

Definition 2: The **conjuction** of p and q, denoted by $p \wedge q$, is the proposition "p and q".

The conjuction $p \wedge q$ is true when both p and q are true, and is false otherwise.

The truth table for the conjuction of two propositions:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjuction

Definition 3: The **disjuction** of p and q, denoted by $p \lor q$, is the proposition "p or q".

The disjuction $p \lor q$ is false when both p and q are false, and is true otherwise.

The truth table for the disjuction of two propositions:

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive or

Definition 4: The **exclusive or** of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true, and is false otherwise.

The truth table for the exclusive or of two propositions:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional Statement

Definition 5: The **conditional statement** of $p \to q$ is the proposition "if p, then q."

The conditional statement $p \to q$ is false when p is true and q is false, true otherwise.

p is called the **hypothesis** (or **antecedent** or **premise**) and q is called the **conclusion** (or **consequence**).

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement

Definition 6: The **biconditional statement** of $p \leftrightarrow q$ is the proposition "p if and only if q."

The biconditional statement $p \leftrightarrow q$ is true when p have the same truth value, and is false otherwise.

Biconditional statements are also called **bi-implications**.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Summary of all Connectives

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \longrightarrow q$	$p \leftrightarrow q$
T	T	F	T	T	F	T	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	F
F	F	T	F	F	F	T	T

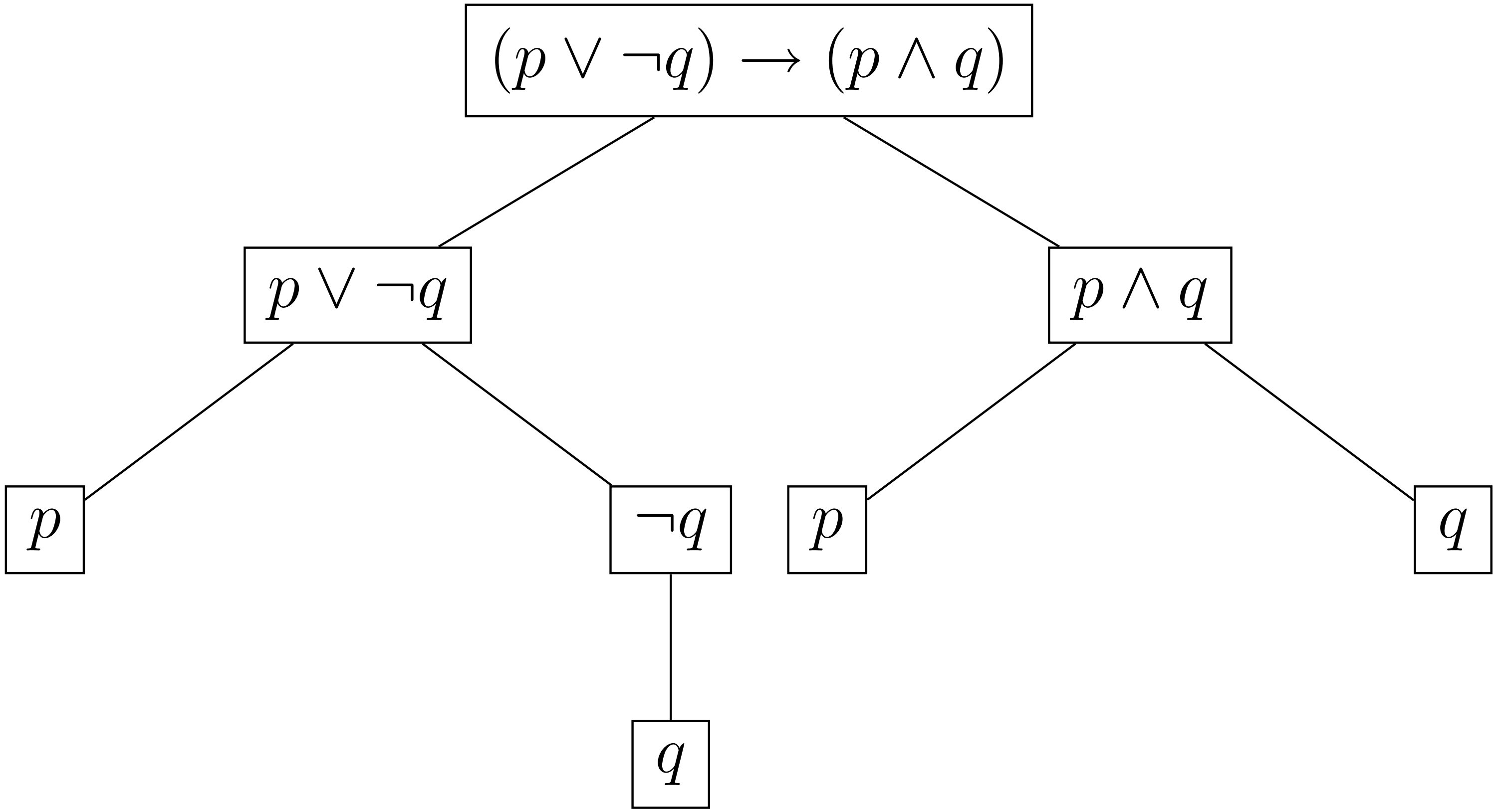
Truth Tables of Compound Propositions

Example 11: Construct the truth table of the compound proposition

$$(p \vee \neg q) \longrightarrow (p \wedge q)\,.$$

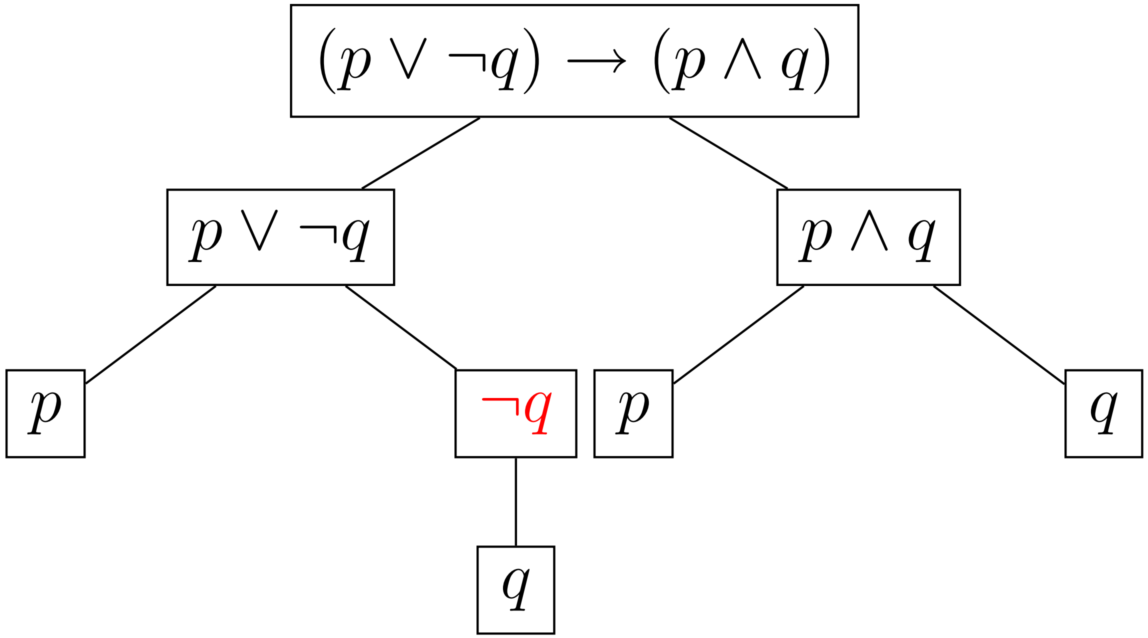
Truth Tables of Compound Propositions

Build the truth table according to the decomposition:



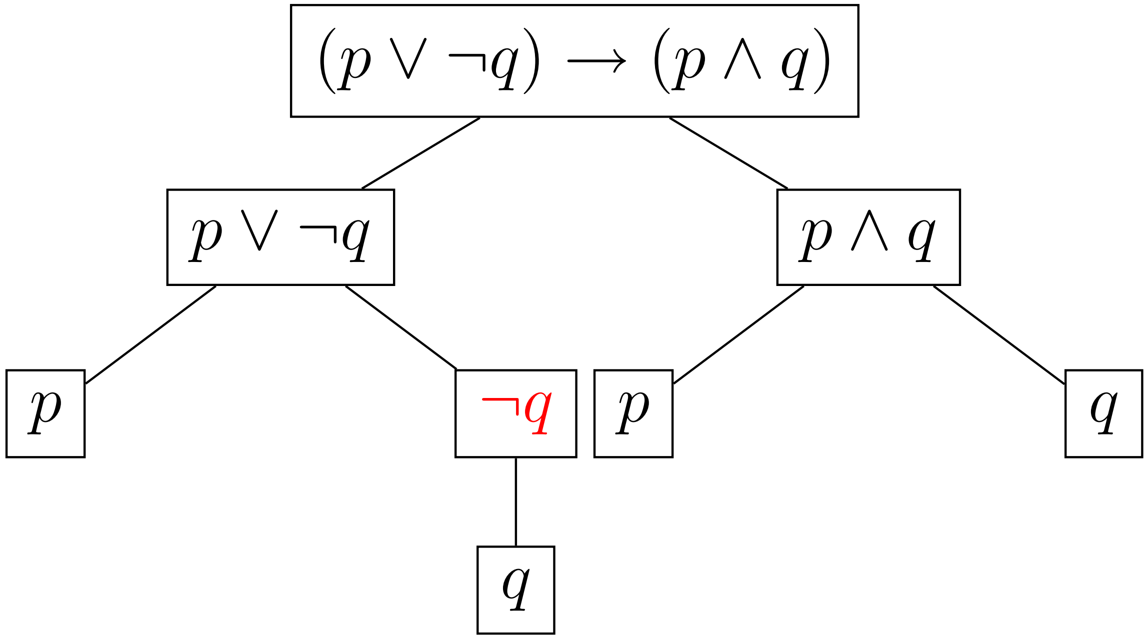
There are four rows in the truth table since the compound proposition involves two propositional variables p and q.

Truth Tables of Compound Propositions



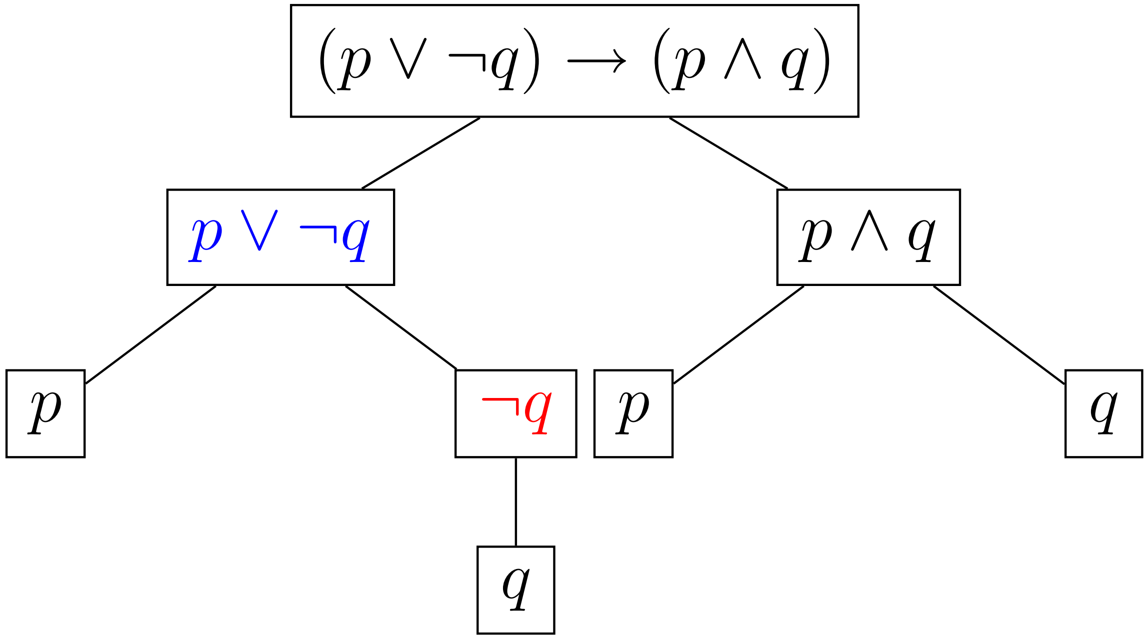
p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

Truth Tables of Compound Propositions



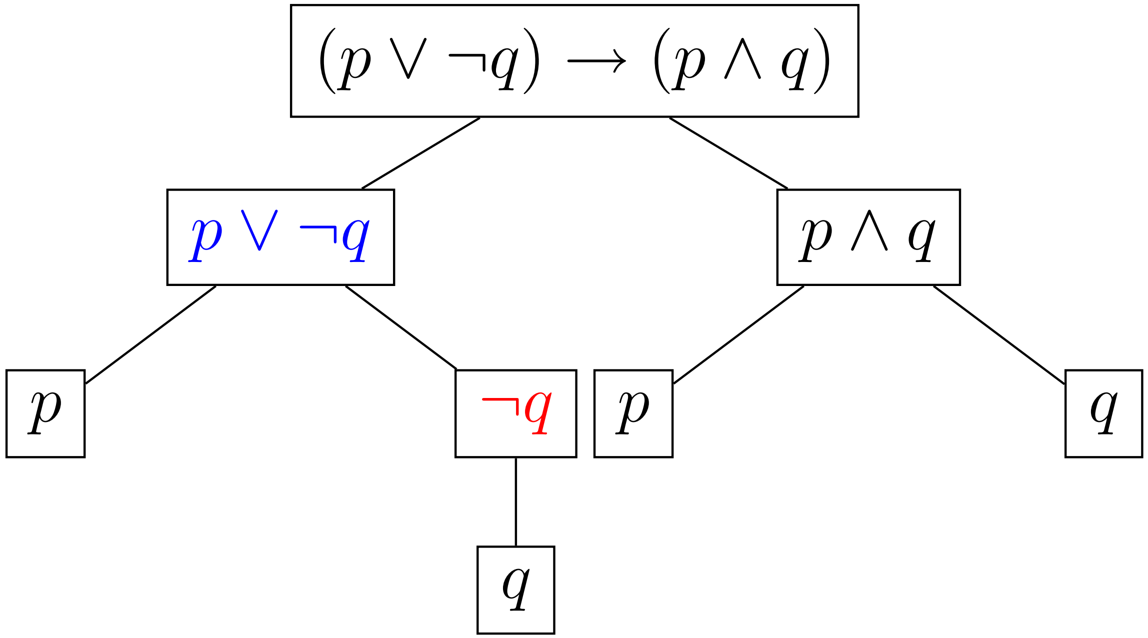
p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Truth Tables of Compound Propositions



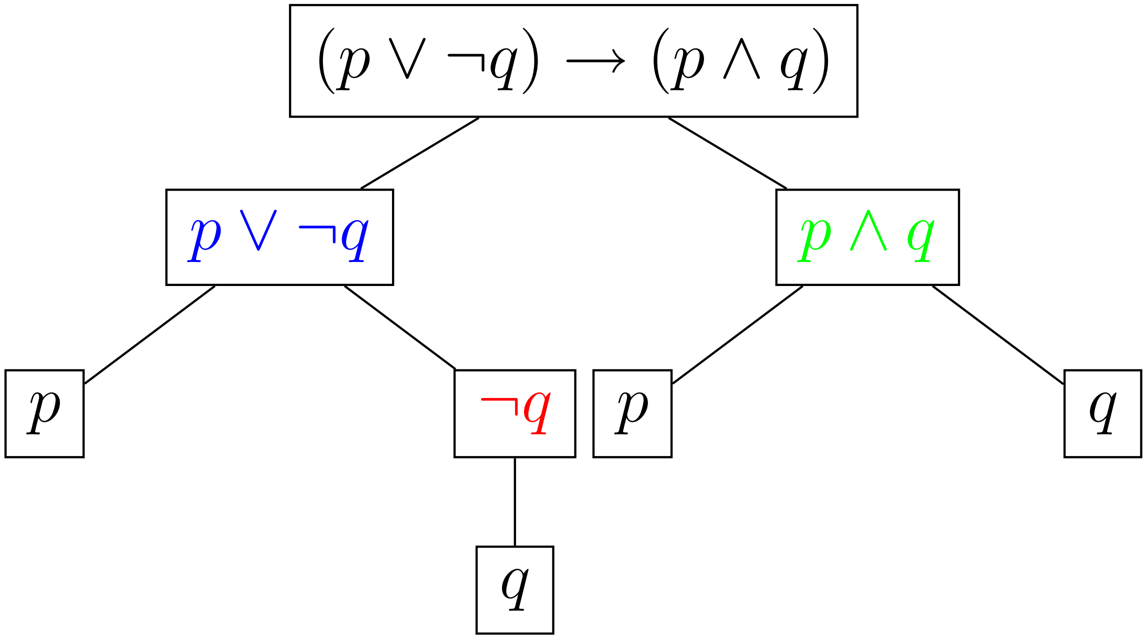
p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Truth Tables of Compound Propositions



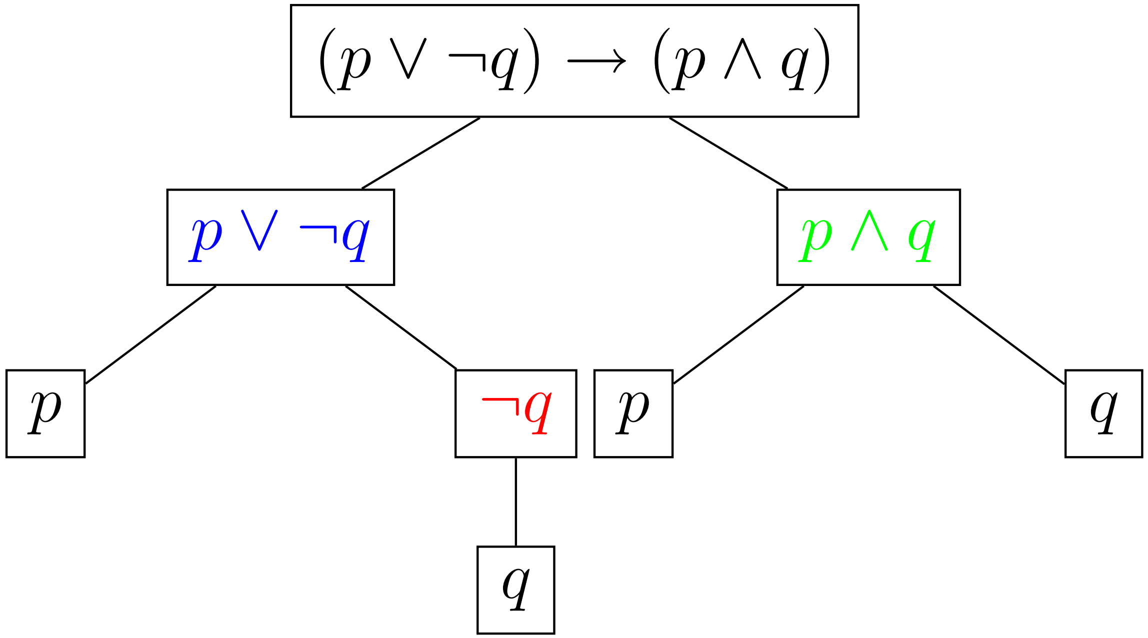
p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T		
T	F	T	T		
F	T	F	F		
F	F	T	T		

Truth Tables of Compound Propositions



p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	
T	F	T	T	F	
F	T	F	F	F	
F	F	T	T	F	

Truth Tables of Compound Propositions



p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Precedence of Logical Operators

Operator	Precedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

$(p \wedge q) \vee \neg r$	$(p \wedge q) \vee (\neg r)$
$p \wedge q \vee r$	$(p \wedge q) \vee r$ rather than $p \wedge (q \vee r)$
$p \vee q \rightarrow r$	$(p \vee q) \rightarrow r$

We will use (redundant) parentheses as in the above expressions to avoid confusion.

Translating English Sentences

"If Greeks are Humans, and Humans are Mortal, then Greeks are Mortal."

$$\big((G \rightarrow H) \wedge (H \rightarrow M)\big) \rightarrow (G \rightarrow M)$$

Translating English Sentences

“Greeks carry Swords or Javelins.”

$$(G \rightarrow S) \vee (G \rightarrow J)$$

True even if a Greek carries both.

Translating English Sentences

"Greeks carry either Bronze or Flint Swords."

$$(G \rightarrow B) \oplus (G \rightarrow F)$$