COT 3100 Discrete Mathematics

Homework 2 Questions

This homework is **due on Friday, February 26th, 2010**.

Problem 1.

Use Laws of Logic and Rules of Inference to justify the following arguments.

(a) (5 pts)

$$\begin{array}{l} \forall x(P(x) \vee Q(x)) \\ \forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x)) \\ \hline \therefore \forall x(\neg R(x) \rightarrow P(x)) \end{array}$$

(b) (5 pts)

$$\begin{array}{l} p \vee q \\ u \wedge r \\ r \rightarrow \neg t \\ (s \vee p) \rightarrow t \\ \hline \therefore q \end{array}$$

Problem 2.

Prove or disprove each of the following statements.

(a) (5 pts) If $n \geq 1$ is a perfect square, then $n + 2$ is not a perfect square.

(b) (5 pts) Consider the following numbers.

$65^{1006} - 8^{2001} + 3^{177}$
$79^{1210} - 9^{2399} + 2^{2001}$
$24^{4491} - 5^{8190} + 7^{1775}$

It is possible to select 2 different numbers from the 3 numbers above such that their product is non-negative.

(c) (5 pts) If a and b are rational numbers, then a^b is also rational.

(d) (5 pts) $\sqrt[3]{2}$ is irrational.

Problem 3.

Let A, B and C be sets and let $P(X)$ be the powerset of set X. Prove or disprove the following statements.

(a) (5 pts) If $A \subseteq (B \cup C)$, then $A \subseteq B$ or $A \subseteq C$.

(b) (5 pts) $(A - C) \cap (C - B) = \emptyset$.

(c) (5 pts) $P(A) - P(B) \subseteq P(A - B)$.