COT 3100 Discrete Mathematics
Homework 1 Questions

January 22, 2010

Problem 1

Section 1.2 #10. Show that each of these conditional statements is a tautology by using truth tables.

(a) (1 point) $[\neg p \wedge (p \vee q)] \rightarrow q$

(b) (1 point) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

(c) (1 point) $[p \wedge (p \rightarrow q)] \rightarrow q$

(d) (1 point) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Section 1.2 #12. Show that each conditional statement above is tautology without using truth tables. Use logical equivalences (P. 24 and P. 25) for this part.

(a) (1 point) $[\neg p \wedge (p \vee q)] \rightarrow q$

(b) (2 points) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

(c) (1 point) $[p \wedge (p \rightarrow q)] \rightarrow q$

(d) (2 points) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Problem 2

Section 1.3 #32. Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not use "It is not that case that ... ")

(a) (2 points) All dogs have fleas.

(b) (2 points) There is a horse that can add.

(c) (2 points) Every koala can climb.

(d) (2 points) No monkey can speak French.

(e) (2 points) There exists a pig that can swim and catch fish.

Problem 3

Section 1.4 #28. (1 point each) Determine the truth value of these statements if the domain of each variable consists of all real numbers.

(a) $\forall x \exists y (x^2 = y)$.

(b) $\forall x \exists y (x = y^2)$.

(c) $\exists x \forall y (xy = 0)$.

(d) $\exists x \exists y (x + y \neq y + x)$.

(e) $\forall x (x \neq 0 \to \exists y (xy = 1))$

(f) $\exists x \forall y (y \neq 0 \to xy = 1)$

(g) $\forall x \exists y (x + y = 1)$

(h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$

(i) $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$

(j) $\forall x \forall y \exists z (z = (x + y)/2)$

Problem 4

Section 1.4 #34. (2 points each) Find a common domain for the variables x, y, and z for which the statement $\forall x \forall y ((x \neq y) \to \forall z ((z = x) \vee (z = y)))$ is true and another domain for which it is false. Justify your answers.

Section 1.4 #46. Determine the truth value of the statement $\exists x \forall y (x \leq y^2)$ if the domain for the variables consists of

(a) (2 points) the positive real numbers.

(b) (2 points) the integers.

(c) (2 points) the nonzero real numbers.