School of Electrical Engineering and Computer Science
University of Central Florida
COT 3100 – Introduction to Discrete Structures
Spring 2010 (PMW)

Name: ____________________ Lab Section #: ______ NID: __________________

Midterm Exam 1 Key

Read carefully and follow the rules below.

1. This exam consists of 9 pages and 3 problems. The space provided should be enough for the answers. Do not add additional pages to the exam.
2. Justify your answers. Make sure that you explain clearly each step in your reasoning.
3. The exam is closed book.

Good luck!!!

1(a)	20	20
1(b)	20	20
2(a)	10	10
2(b)	10	10
2(c)	10	10
3(a)	10	10
3(b)	10	10
3(c)	10	10
Total	100	100

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Problem 1

(a) (20 points) Show that the statement $[(q \to p) \wedge (\neg r \to \neg p)] \to (q \to r)$ is a tautology by using a truth table.

p	q	r	$q \to p$	$\neg r$	$\neg p$	$\neg r \to \neg p$	$(q \to r)$
T	T	T	T	F	F	T	T
T	T	F	T	T	F	F	F
T	F	T	T	F	F	T	T
T	F	F	T	T	F	F	T
F	T	T	F	F	T	T	T
F	T	F	F	T	T	T	F
F	F	T	T	F	T	T	T
F	F	F	T	T	T	T	T

$(q \to p) \wedge (\neg r \to \neg p)$	$[(q \to p) \wedge (\neg r \to \neg p)] \to (q \to r)$
T	T
F	T
T	T
F	T
F	T
F	T
T	T
T	T

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(b) (20 points) Show that $[(r \vee p) \wedge (\neg r \vee q)] \rightarrow (p \vee q)$ is a tautology using logical equivalences. Be sure to present all steps with the name of the laws you are applying at each step.

$[(r \vee p) \wedge (\neg r \vee q)] \rightarrow (p \vee q)$	
$\equiv \neg[(r \vee p) \wedge (\neg r \vee q)] \vee (p \vee q)$	(Rule of Implication)
$\equiv [\neg(r \vee p) \vee \neg(\neg r \vee q)] \vee (p \vee q)$	(DeMorgan's Law)
$\equiv [(\neg r \wedge \neg p) \vee (r \wedge \neg q)] \vee (p \vee q)$	(DeMorgan's Law / Law of Double Negation)
$\equiv (\neg r \wedge \neg p) \vee (r \wedge \neg q) \vee p \vee q$	(Associative Law)
$\equiv (p \vee (\neg r \wedge \neg p)) \vee (q \vee (r \wedge \neg q))$	(Commutative and Associative Laws)
$\equiv ((p \vee \neg r) \wedge (p \vee \neg p)) \vee ((q \vee r) \wedge (q \vee \neg q))$	(Distributive Law)
$\equiv ((p \vee \neg r) \wedge \mathrm{T}) \vee ((q \vee r) \wedge \mathrm{T})$	(Negation Law)
$\equiv (p \vee \neg r) \vee (q \vee r)$	(Identity Law)
$\equiv (\neg r \vee r) \vee (p \vee q)$	(Commutative and Associative Laws)
$\equiv \mathrm{T} \vee (p \vee q)$	(Negation Law)
$\equiv \mathrm{T}$	(Domination Law)

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Problem 2

Write the negation of each of these logical expressions, such that there are no negation signs to the left of a quantifier or a compound expression. In the final result, negation signs may only appear in front of single variables or predicates. Show your steps, but you do not need to name the laws you use for each step.

(a) (10 points) $p \to (p \to q)$

$\neg(p \to (p \to q))$
$\equiv p \land \neg(p \to q)$
$\equiv p \land (p \land \neg q)$
$\equiv p \land \neg q$

(b) (10 points) $\forall x \exists y \forall z ((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y)))$

$$
\begin{aligned}
&\neg(\forall x \exists y \forall z ((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y)))) \\
&\equiv \exists x \neg(\exists y \forall z ((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y)))) \\
&\equiv \exists x \forall y \neg(\forall z ((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y)))) \\
&\equiv \exists x \forall y \exists z \neg((x \neq z) \rightarrow \neg(T(x,y) \wedge T(z,y))) \\
&\equiv \exists x \forall y \exists z (x \neq z) \wedge T(x,y) \wedge T(z,y)
\end{aligned}
$$

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(c) (10 points) $\forall\epsilon\exists\delta\forall x((0 < |x - a| < \delta) \to (|f(x) - L| < \epsilon))$

$$
\begin{aligned}
&\neg(\forall\epsilon\exists\delta\forall x((0 < |x - a| < \delta) \to (|f(x) - L| < \epsilon)))\\
&\equiv \exists\epsilon\forall\delta\exists x\neg((0 < |x - a| < \delta) \to (|f(x) - L| < \epsilon))\\
&\equiv \exists\epsilon\forall\delta\exists x(0 < |x - a| < \delta) \wedge \neg(|f(x) - L| < \epsilon)\\
&\equiv \exists\epsilon\forall\delta\exists x(0 < |x - a| < \delta) \wedge (|f(x) - L| \geq \epsilon)
\end{aligned}
$$

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Problem 3

Determine the truth value of these statements if the domain of each variable consists of all *positive integers*. Justify your answers.

(a) (10 points) $\forall x \exists y (x^2 = y)$.

True. Let x be given. Then, let $y = x^2$. If x is a positive integer, so is x^2. Thus, y is also a positive integer.

(b) (10 points) $\exists x \forall y (x \leq y)$.

True. Let $x = 1$. 1 is the smallest positive integer.

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(c) (10 points) $\forall x \exists y \exists z ((y > 1) \land (z > 1) \land (x = yz))$

False. To show that the statement is false, consider the negation $\exists x \forall y \forall z ((y \leq 1) \lor (z \leq 1) \lor (x \neq yz)) \equiv \exists x \forall y \forall z (((y > 1) \land (z > 1)) \to (x \neq yz))$. To show that the negation is a tautology, let $x = 1$, then let y, z be given. If $y > 1$ and $z > 1$, then $yz > 1$. So, $x \neq yz$.

In fact, the negation can be shown true for any prime number x, in addition to $x = 1$.