

COT 3100H Final Exam - Part B - 100 pts (12/5/2024)

Last Name: _____ , **First Name:** _____

1) (8 pts) Please complete the truth table below.

p	q	r	$p \vee q$	$\bar{r} \rightarrow \bar{q}$	$(p \vee q) \rightarrow (\bar{r} \rightarrow \bar{q})$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (10 pts) Prove OR disprove the following assertion for finite sets A, B and C. First state whether the assertion is true or false, followed by a detailed justification of your answer.

$$\text{If } A - B \subseteq A - C, \text{ then } C \subseteq A \cap B.$$

3) (10 pts) Find all ordered pairs of integers, (x, y) , that satisfy the equation $277x + 122y = 5$. Put a box around your final answer.

4) (10 pts) **Prove via induction on n** , that for all positive integers n , $\sum_{i=1}^n \frac{i}{i+1} \leq \frac{n^2}{n+1}$.

5) (10 pts) **Prove via induction on n ,** that for all integers $n \geq 5$, $\binom{2n}{n} < 2^{2n-2}$.

6) (10 pts) How many permutations of the letters AAABBBCCC do not have any substring of length 3 of the same letter? (For example, you should count AABBBCCABC but you should not count AABBBABCCC or CBAABBBCC.) First work out your answer in permutations, combinations, etc. but since you have a calculator, simplify your expression to a single integer. (Hint: Think Inclusion/Exclusion Principle!)

7) (10 pts) There are 100 chips in a bin. Each chip is colored with exactly one color: red or black, and there are more red chips than black chips. If someone selects two chips at random without replacement, the probability that they receive 2 chips of the same exact color is precisely 50%. How many red chips are there in the bin?

8) (10 pts) Let X be a continuous random variable defined by the probability density function below:

$$p(x) = x^3 + cx, 0 \leq x \leq 1$$
$$p(x) = 0, \text{otherwise}$$

(a) (3 pts) Find the value of c . Please express your answer as a fraction in lowest terms.

(b) (3 pts) Determine the probability that $0 \leq X \leq \frac{1}{2}$. Please express your answer as a fraction in lowest terms.

(c) (4 pts) Determine the value of $E(X)$. Please express your answer as a fraction in lowest terms.

9) (5 pts) Given that $2^{16} + 1$ is a prime number and $2^{32} + 1$ is the product of two prime numbers (641, and a much larger one), how many divisors does $2^{64} - 1$ have?

10) (5 pts) If we were to take $1000!$ (where 1000 is represented in base 10), and then convert it to base 8, how many zeroes would this converted number have at the end of it?

11) (5 pts) An infinite geometric series, $a_1, a_2, a_3, \dots$, has a sum of 60. The sum of the even indexed terms, $a_2, a_4, a_6, \dots$, is 24. Determine a_1 as well as the common ratio for the series.

$a_1 =$ _____ , $r =$ _____

12) (5 pts) Steve takes a round trip from Orlando to Jacksonville. For the whole round trip, his average speed was 48 miles per hour. On the way there he averaged 60 miles per hour. What was his average speed on the way back?

13) (2 pts) Moana 2 recently opened in theaters across the US.
How many Moana movies have been made so far?

Scratch Page – Please clearly mark any work you would like graded on this page.