

COT 3100 Final Exam - Part A (Relations, Functions) - 25 pts (12/3/2024)

Last Name: _____ , **First Name:** _____

1) (5 pts) How many anti-symmetric relations over the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ contain the ordered pairs $(1, 1)$, $(2, 3)$, $(6, 4)$, $(7, 7)$, and $(8, 3)$? Please leave your answer as a product of terms, each of which is written in exponential form.

2) (6 pts) Let $R = \{ (a, b) \mid \exists c \in \mathbb{Z}^+ \text{ such that } ab = c^2 \}$ be a relation defined over the positive integers. Prove that R is transitive. (You may use the fact that if a rational number squared is an integer, then that rational number itself is also an integer.)

3) (8 pts) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions where f is a surjective function and g is an injective function.

(a) Prove or disprove: the composition function, $g \circ f$ is a surjective function.

(b) Prove or disprove: the composition function, $g \circ f$ is an injective function.

For each part, clearly state whether the assertion is true or not, followed by a proper justification.

4) (6 pts) Let r , s and t be roots of the cubic polynomial $f(x) = 3x^3 - 5x^2 + 7x - 9$. What is the cubic equation, with **leading coefficient 9**, with the roots $\frac{1}{r}$, $\frac{1}{s}$, and $\frac{1}{t}$?

Scratch Page – Please clearly mark any work on this page you would like graded.