COT 3100 Quiz #3: Counting, Probability (Week of Apr 10) – R, F Version Solutions

1) (5 pts) How many permutations are there of the letters in the word "SMUSHROOMS"? Please leave your answer in factorials, combinations and/or powers.

There are 10 letters. The repeated letters (and their frequencies) are: S = 3, M = 2, O = 2. Using the permutation formula, it follows that the answer is $\frac{10!}{3!2!2!}$.

Grading: (Based on answer only) 2 pts for numerator, 1 pt to divide by something, 2 pts for the denominator.

2) (7 pts) A faulty car odometer proceeds from digit 5 to digit 8, always skipping digits 6 and 7, regardless of position. For example, after traveling one mile, the odometer changed from 003859 to 003880. If the odometer now reads 003459, how many miles has the car actually traveled? (Note: $8^3 = 512$.)

There are 8 possible settings for each digit on the odometer: 0, 1, 2, 3, 4, 5, 8 and 9. Thus, the odometer is a number in base 8 except when 8 is shown its actual value is 6 and when 9 is shown, its actual value is 7. It follows that the desired answer is $3 \times 8^3 + 4 \times 8^2 + 5 \times 8^1 + 7 =$

$3 \times 512 + 4 \times 64 + 40 + 7 =$
$1536 + 256 + 47 =$
$1792 + 47 =$
1839.

Grading: Full credit for the correct answer as long as the work seems to support the answer (most can't do this in their head, so if an answer just appears, they copied)

Stating base 8 observation – 2 pts
Writing out base 8 conversion – 2 pts
Arithmetic to get answer – 3 pts
Give 6/7 if they answer 1840 or 1841.

3) (5 pts) How many non-negative integer solutions are there to the equation

$$a + b + c + d + e + f \leq 30?$$

Please leave your answer in factorials, combinations and/or powers.

Add a slack variable g. The number of non-negative integer solutions to a+b+c+d+e+f+g=30 is the same as the number of non-negative integer solutions to the equation above (set g to the "leftover" from 30). It follows that there are $\binom{30+7-1}{7-1} = \binom{\mathbf{36}}{\mathbf{6}}$.

Grading: 5 pts – correct answer
4 pts – for C(35, 5) or C(35, 6)
2 pts – for C(30, 5) or C(30, 6)
1 pt – for 30^6
0 pts - otherwise

4) (8 pts) A bag of prizes contains 40% transformer toys and 60% pez dispensers. Unfortunately, 10% of the transformer toys are defective and 6% of the pez dispensers are defective. An item is selected at random from the bag and is defective. What is the probability that the item is a transformer toy? **Please answer in the form of a fraction reduced to lowest terms.**

Let A be the event a transformer toy is chosen from the bag. Let $\bar{A}$ be the event a pez dispenser is chosen. Let B be the event that an item is defective.

The given information is as follows: $p(A) = .4$, $p(\bar{A}) = .6$ $p(B|A) = .1$, $p(B|\bar{A}) = .06$.

$p(B) = p(A) \times p(B|A) + p(\bar{A}) \times p(B|\bar{A}) = .4 \times .1 + .6 \times .06 = .04 + .036 = .076.$

We desire the value of $p(A|B)$. $p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(A) \times p(B|A)}{.076} = \frac{.04}{.076} = \frac{\mathbf{40}}{\mathbf{76}} = \frac{\mathbf{10}}{\mathbf{19}}$

Grading: 2 pts to translate given values into formal symbols (for partial credit)
2 pts to obtain p(B)
2 pts to obtain p(A and B)
1 pt to get the correct answer but not in a reduced fraction
1 pt to express correctly as 10/19.