

COT 3100 Quiz #3: Counting, Probability (Week of Apr 10) – M, T, W Version Solution

1) (5 pts) How many permutations are there of the letters in the word "SPECIFICITY"? Please leave your answer in factorials, combinations and/or powers.

There are 11 letters. The repeated letters (and their frequencies) are: C = 2, and I = 3. Using the permutation formula, it follows that the answer is $\frac{11!}{2!3!}$.

Grading: (Based on answer only) 2 pts for numerator, 1 pt to divide by something, 1 pt for each term in the denominator (full credit only if answer is correct).

2) (7 pts) How many ordered four-tuplets of integers, (a, b, c, d), are there with $1 \leq a < b < c < d \leq 30$? Please leave your answer in factorials, combinations and/or powers.

Each choice of 4 integers out of 30 (1 through 30) results in a single, unique solution for a, b, c and d. In addition, each solution for (a, b, c, d) maps to a unique combination of 4 integers chosen out of 30. Since the one to one correspondence has been established, it follows that the number of solutions is the number of combinations of 4 items out of 30, which, by definition, is $\binom{30}{4}$.

Grading: Full credit for the answer, no explanation needed. Give partial credit for the following answers:

$30^4 - 2$ pts

$30 \times 29 \times 28 \times 27 - 3$ pts

Other answers – 0 or 1 pt (grader discretion)

3) (5 pts) A line passes through A(73, 35) and B(178, 230). How many **other points** with integer coordinates are on the line segment that connects A to B?

The change in x for the line segment is $178 - 73 = 105$.

The change in y for the line segment is $230 - 35 = 195$.

Let's compute $\gcd(195, 105)$:

$$195 = 1 \times 105 + 90$$

$$105 = 1 \times 90 + \mathbf{15}$$

$$90 = 6 \times 15.$$

Thus, lattice points on the line segment take the form $x = 73 + 7a$, $y = 35 + 13a$, for $a = 0, 1, \dots, 15$. It follows that there are $16 - 2 = \mathbf{14}$ other lattice points on the line segment.

Grading: determine dx – 1 pt

determine dy – 1 pt

determine gcd – 1 pt (any way is fine)

reasoning out answer – 2 pts

note: award 4 out of 5 for either 15 or 16 as an answer.

4) (8 pts) A bag of prizes contains 20% transformer toys and 80% pez dispensers. Unfortunately, 15% of the transformer toys are defective and 5% of the pez dispensers are defective. An item is selected at random from the bag and is defective. What is the probability that the item is a transformer toy? **Please answer in the form of a fraction reduced to lowest terms.**

Let A be the event a transformer toy is chosen from the bag. Let \bar{A} be the event a pez dispenser is chosen. Let B be the event that an item is defective.

The given information is as follows: $p(A) = .2$, $p(\bar{A}) = .8$, $p(B|A) = .15$, $p(B|\bar{A}) = .05$.

$$p(B) = p(A) \times p(B|A) + p(\bar{A}) \times p(B|\bar{A}) = .2 \times .15 + .8 \times .05 = .03 + .04 = .07.$$

$$\text{We desire the value of } p(A|B). \quad p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(A) \times p(B|A)}{.07} = \frac{.03}{.07} = \frac{3}{7}$$

Grading: 2 pts to translate given values into formal symbols (for partial credit)

2 pts to obtain p(B)

2 pts to obtain p(A and B)

1 pt to get the correct answer but not in a reduced fraction

1 pt to express correctly as 3/7.