COT 3100 Quiz #2: d = rt, logs (Week of Mar 20, 2023) – R, F Version Solutions

1) (8 pts) Let x and y be positive integers such that xy + 3x + 4y = 209, with x < 10 and y < 20, with proof (without exhaustive search), find x and y. (Note: As a hint, when you solve this problem, you should run into factoring 221 = 13 x 17.)

xy + 3x + 4y = 209
xy + 3x + 4y + 12 = 209 + 12
(x + 4)(y + 3) = 221 = 13 x 17 = 1 x 221

It follows that x + 4 = 1, 13, 17 or 221

Because we know x is positive and less than 10, the only possible solution is x + 4 = 13 → **x = 9.**
It follows that y + 3 = 17, so **y = 14.**

Note: An alternate solution solves for x: $x = -4 + \frac{221}{y+3}$, via isolating x and polynomial division, and then cites that y + 3 must be 1, 13, 17 or 221, and proceeds from there.

Grading: 2 pts to add 12 to both sides, 2 pts to factor LHS, 2 pts to consider 4 options, 1 pt to narrow down options to 1, 1 pt to give the final answers for both x and y.

2) (7 pts) Josie buys 3 apples and 5 oranges for $5.08. Later, she buys 2 apples and 1 orange for $1.52. How much would Josie have spent if she bought 1 apple and 2 oranges?

Let a be the cost of an apple and b be the cost of an orange. Then, we have:

3a + 5b = 5.08 → 6a + 10b = 10.16
2a + b = 1.52 → 6a + 3b = 4.56

7b = 5.60
b = 0.80, so 2a + .8 = 1.52 → 2a = .72 → a = 0.36

The cost of 1 apple and 2 oranges is $0.36 + 2($0.80) = **$1.96**

Grading: 1 pt create variables for apple, orange prices
1 pt write down first equation
1 pt write down second equation
3 pts solve system for both a and b
1 pt answer given query

3) (10 pts) Let r and s be roots of the quadratic equation $x^2 - 6x + 17 = 0$. What is the quadratic equation with leading coefficient **<u>17</u>** which has the roots $(r + \frac{1}{s})$ and $(s + \frac{1}{r})$? (Note: Once you start working out the problem, you'll see why I asked for a leading coefficient of 17…)

Using the given information, we have that r + s = 6, rs = 17.

To find the desired equation, we must solve for:

(a) $r + \frac{1}{s} + s + \frac{1}{r} = (r + s) + \frac{r+s}{rs} = 6 + \frac{6}{17} = \frac{108}{17}$

(b) $\left(r + \frac{1}{s}\right)\left(s + \frac{1}{r}\right) = rs + 1 + 1 + \frac{1}{rs} = 17 + 2 + \frac{1}{17} = \frac{324}{17}$

It follows that a quadratic with leading coefficient of 1 which satisfies the query is

$$x^2 - \frac{108}{17}x + \frac{324}{17} = 0$$

If we multiply this through by 17, we get the desired quadratic (with the same roots) with a leading coefficient of 17:

$$17x^2 - 108x + 324 = 0$$

Grading: 1 pt write down r + s
1 pt write down rs
3 pts solve for $r + \frac{1}{s} + s + \frac{1}{r}$
3 pts solve for $\left(r + \frac{1}{s}\right)\left(s + \frac{1}{r}\right)$
2 pts final answer

If students can solve this by getting r and s individually and get it correctly, go ahead and give them full credit. It's not really that fun!