COT 3100 Quiz #2: d = rt, logs (Week of Mar 20, 2023) – M, T, W Version Solutions

1) (8 pts) How many zeroes does 200! end in, when represented in base 24? (Note: Be careful, consider both 2 and 3…)

$24 = 2^3 \times 3$.

Thus, we need to calculate the number of times 2 divides evenly into 200! and the number of times that 3 divides evenly into 200! Here are both calculations using the algorithm shown in class:

2 | 200
2 | 100
2 | 50
2 | 25
2 | 12
2 | 6
2 | 3
2 | 1
Total = 100 + 50 + 25 + 12 + 6 + 3 + 1 = 197

3 | 200
3 | 66
3 | 22
3 | 7
3 | 2
Total = 66 + 22 + 7 + 2 = 97

It follows that the number of times 2^3 divides evenly into 200! is $\left\lfloor \frac{197}{3} \right\rfloor = 65$. Since this is less than 97, the number of times 3 divides evenly into 200!, it follows that the number of zeroes at the end of 200! in base 24 is **65.**

Grading: 3 pts calculate # of times 2 divides into 200!
3 pts calculate # of times 3 divides into 200!
1 pt to determine # of times 2^3 divides into 200!
1 pt to arrive at the answer.

2) (7 pts) Stan starts with a mixture of acid and water which is a total of X ounces. When he adds 2 ounces of acid to the mixture, it becomes a solution that is 50% acid. If an additional 5 ounces of water is added to this 50% acid mixture, the result is a solution that is $33\frac{1}{3}$% acid. What is the value of X?

Let amount of acid in the original mixture be A. Using the given information, we have $\frac{A+2}{X+2} = \frac{1}{2}$. Using the second piece of information, we have $\frac{A+2}{X+7} = \frac{1}{3}$. Cross multiply both equations to get linear equations in A and X:

$2A + 4 = X + 2$
$X = 2A + 2$, to substitute

$3A + 6 = X + 7$
$3A + 6 = 2A + 2 + 7$
$A = 3$, thus $X = 2(3) + 2 = \mathbf{8}$

Grading: 1 pt create variable for acid, 1 pt set up first equation, 1 pt set up second equation, 4 pts to solve the system for X. (Take off 1 point if the answer given is for A.)

3) (10 pts) Let r and s be roots of the quadratic equation $x^2 - 4x + 10 = 0$. What is the quadratic equation with leading coefficient one which has the roots r^2s and rs^2?

Using the given information, we have that $r + s = 4$, $rs = 10$.

To find the desired equation, we must solve for:

(a) $r^2s + rs^2 = rs(r + s) = 10(4) = 40$

(b) $(r^2s)(rs^2) = r^3s^3 = (rs)^3 = 10^3 = 1000$

It follows that the desired quadratic is $x^2 - 40x + 1000 = 0$.

Grading: 1 pt write down r + s
1 pt write down rs
3 pts solve for $r^2s + rs^2$
3 pts solve for $(r^2s)(rs^2)$
2 pts final answer

If students can solve this by getting r and s individually and get it correctly, go ahead and give them full credit. It's not really that fun!