COT 3100 Quiz #1: d = rt, logs (Week of Feb 6, 2023) – R, F Version Solutions

1) (8 pts) Trusty Tortoise and Roger Rabbit are taking a (same) journey. Trusty Tortoise travels at an average speed of 2 miles an hour and never stops on the journey. Roger Rabbit will spend 10 minutes traveling at an average speed of 5 miles an hour before taking a 20 minute break, and alternating this pattern. Find any distance, D_1, such that Trusty Tortoise wins the race, stating, with proof, the amount of time both Trusty Tortoise and Roger Rabbit take to traverse that distance. (Express both times in HR:MIN:SEC format.)

If the Tortoise is ever going to be ahead, it will be at the 30 minute mark, at the end of the Rabbit's rest. Let's work out where both are 30 minutes = $\frac{1}{2}$ hour into the journey:

Trusty Tortoise will travel $1 mph \times \frac{1}{2} hr = 1\ mile$.

In $\frac{1}{2}$ hour, Roger Rabbit will only travel $5\ mph \times \frac{1}{6} hr = \frac{5}{6}\ miles$, since he's only traveling for 10 minutes of the journey, which is $\frac{1}{6}$ hour.

This means, that when Roger starts after his rest, he has $1 - \frac{5}{6} = \frac{1}{6}\ miles$ left to travel. It will take him $\frac{1/6 mile}{5mph} = \frac{1}{30} hr = \frac{60}{30} min = 2\ minutes$ to "finish" the 1 mile race.

D_1 = **1 mile** , Tortoise Time = **30 minutes** , Rabbit Time = **32 minutes**

Grading: 2 points for any valid distance, 3 pts for the Tortoise's time for their chosen distance, 3 pts for the Rabbit's time for their chosen distance, given partial as you see fit.

2) (8 pts) An arithmetic sequence of 30 terms has a sum of 765. If the common difference and first term of the sequence are both positive integers, what are the values of the first term and common difference? (Please put your answer on the given slots on the back of this page.

Let a_1 be the first term and d be the common difference of the sequence. The thirtieth term of the sequence is $a_1 + 29d$. It follows that the sum of the sequence is $\frac{a_1+(a_1+29d)}{2} \times 30 = (2a_1 + 29d) \times 15$.

Set this equal to 765: $(2a_1 + 29d) \times 15 = 765 \rightarrow 2a_1 + 29d = \frac{765}{15} = 51$.

If d is a positive integer, it must be 1, because if d = 2, that forces a to be a negative integer, which isn't true. Thus, set d = 1 and solve for a: $2a_1 = 51 - 29(1) = 22 \rightarrow a_1 = 11$.

Thus, the first term is **$a_1 = 11$** and the common difference **d = 1.**

Grading: 2 pts formula for a_{30}, 2 pts formula for sum, 1 pt divide by 15, 2 pts reason out d, 1 pt get a from d.

3) (9 pts) Find the value of the following sum:

$$log_{1024}2 + log_{1024}4 + log_{1024}8 + log_{1024}16 + \cdots + log_{1024}(2^{20})$$

Note: $2^{10} = 1024$. Put a box around your final answer.

Rewrite each value in each log as a power of 1024:

$$log_{1024}1024^{1/10} + log_{1024}1024^{2/10} + log_{1024}1024^{3/10} + log_{1024}1024^{4/10} + \cdots + log_{1024}(1024^{20/10})$$

Then, by definition of log and the arithmetic sum formula, we have:

$$\frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \cdots \frac{20}{10} = \frac{20 \times 21}{2 \times 10} = \mathbf{21}$$

Grading: 4 pts to rewrite logs in a way that uncovers the arithmetic series, 5 pts to get the correct series sum, grader decides partial.