COT 3100 Quiz #1: d = rt, logs (Week of Feb 6, 2023) – M, T, W Version Solution

1) (8 pts) Trusty Tortoise and Roger Rabbit are taking a (same) journey. Trusty Tortoise travels at an average speed of 3 miles and hour and never stops on the journey. Roger Rabbit will spend 10 minutes traveling at an average speed of 8 miles an hour before taking a 20 minute break, and alternating this pattern. Find any distance, D_1, such that Trusty Tortoise wins the race, stating, with proof, the amount of time both Trusty Tortoise and Roger Rabbit take to traverse that distance. (Express both times in HR:MIN:SEC format.)

If the Tortoise is ever going to be ahead, it will be at the 30 minute mark, at the end of the Rabbit's rest. Let's work out where both are 30 minutes = $\frac{1}{2}$ hour into the journey:

Trusty Tortoise will travel $3mph \times \frac{1}{2}hr = \frac{3}{2}\ miles.$

In $\frac{1}{2}$ hour, Roger Rabbit will only travel $8\ mph \times \frac{1}{6}hr = \frac{4}{3}\ miles$, since he's only traveling for 10 minutes of the journey, which is $\frac{1}{6}$ hour.

This means, that when Roger starts after his rest, he has $\frac{3}{2} - \frac{4}{3} = \frac{9-8}{6} = \frac{1}{6}\ miles$ left to travel. It will take him $\frac{1/6 mile}{8mph} = \frac{1}{48}hr = \frac{60}{48}min = \frac{5}{4}min = 1 minute, 15 seconds$

D_1 = **3/2 miles** , Tortoise Time = **30 min** , Rabbit Time = **31 min and 15 sec**

Grading: 2 points for any valid distance, 3 pts for the Tortoise's time for their chosen distance, 3 pts for the Rabbit's time for their chosen distance, given partial as you see fit.

2) (8 pts) Let S be an arithmetic sequence of 50 terms with a first term of a_1 and a common difference of 3. Let T be an arithmetic sequence of 50 terms with a first term of a_1 and a common difference of 5. Let sum(X) represent the sum of the terms in the arithmetic sequence X. What is the value of sum(T) – sum(S)? (Put a box around your final answer.)

T = $a_1 + (a_1 + 5) + \ldots + (a_1 + 49(5))$
-S = $a_1 + (a_1 + 3) + \ldots + (a_1 + 49(3))$

T-S = $0 + 2 + 4 + 6 \ldots + 98 = [(0 + 98)/2] \times 50 = 49 \times 50 =$ **2450**

Grading: 4 pts to get to the sum that is the difference of the two sequences (can be done in different ways), 4 pts to evaluate the sum. Give partial credit as you see fit.

3) (9 pts) The following relationship holds between positive real numbers x and y:

$$log_{\sqrt{2}}x + log_4 x^3 = log_8 y$$

In order for this equation to hold, we can express y in the form $y = x^{\frac{a}{b}}$, where a and b are both positive integers with gcd(a, b) = 1. (The exponent is a fraction in lowest terms.) Find the fraction, $\frac{a}{b}$, for which $y = x^{\frac{a}{b}}$. (Put a box around your final answer.)

Change the base to 2:

$$\frac{log_2 x}{log_2\sqrt{2}} + \frac{log_2 x^3}{log_2 4} = \frac{log_2 y}{log_2 8}$$

$$\frac{log_2 x}{\frac{1}{2}} + \frac{3log_2 x}{2} = \frac{log_2 y}{3}$$

$$2log_2 x + \frac{3log_2 x}{2} = \frac{log_2 y}{3}$$

$$\frac{7log_2 x}{2} = \frac{log_2 y}{3}$$

$$21log_2 x = 2log_2 y$$

$$\frac{21log_2 x}{2} = log_2 y$$

$$log_2 y = log_2 x^{21/2}$$

It follows that $y = x^{\frac{21}{2}}$. Thus, the desired fraction is $\frac{a}{b} = \frac{21}{2}$.