COT 3100H Final Exam - Part A (Recitation Topics) - 30 pts

Date: 5/3/2021
Start Time: 10:00 am EDT
End Time: 10:40 am EDT

1) (10 pts) A course grade is comprised of 5 Quizzes, 10 Homework Assignments and 4 Exams. The quizzes are worth 30% of the course grade, the homework assignments are worth 10% of the course grade and the exams are worth the remaining 60% of the course grade. Each of the quizzes is out of 60 points, each of the homework assignments is out of 20 points and each of the exams is out of 100 points. Each quiz is worth the same amount of the course grade, each homework assignment is worth the same amount of the course grade and each exam is worth the same amount of the course grade. Note that a quiz, homework assignment and exam are each worth different amounts of the course grade.

Thus far in the semester, 3 quizzes have been administered, 7 homework assignments have been given and 1 exam has been given.

Let **X** represent the current grade (as a percentage) reported in Webcourses (which isn't accurate), and
Let **Y** represent the actual current grade (as a percentage) of the student.

With proof, determine the **maximum** possible value of X - Y. (Note: a "greedy" strategy without any Calculus is sufficient for the maximization.) What percentages in each category achieve this maximum difference? (Note: there are no absolute value bars around X - Y.)

2) (10 pts) Let A be an arithmetic sequence of 30 terms, with a first term of 2 with a common difference of 3. Let B be an infinite geometric sequence with the same first term as A AND the same sum as A. What is the common ratio of B? **Express your answer as a fraction in lowest terms. Feel free at the very end to multiply two numbers on your calculator to speed up computation, but the rest of the work must be done by hand.**

3) (10 pts) Determine the following sum, simplifying your result to a single value without the use of a calculator. (Note: $2^{10} = 1024$.)

$$\sum_{i=1}^{100} log_{1024}(4^i)$$

COT 3100H Final Exam - Part B (Logic, Sets, Relations, Functions) - 30 pts

Date: 5/3/2021
Start Time: 10:40 am EDT
End Time: 11:20 am EDT

1) (6 pts) Prove or disprove the following statement over the universe of positive real numbers:

$$\forall x \exists y [3(x - y) = 2xy + 1]$$

2) (6 pts) Let A and B be sets such that $|A - B| = 2$, $|B - A| = 3$ and $|A \cap B| = 4$. Determine the value of $|\wp(A \cup B)|$. Recall that $\wp(X)$ denotes the power set of the set X.

3) (10 pts) Define the following relation over the set of positive integers:

R = { (a, b) | a + b is a prime number }

With proof determine if R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and € transitive. For each answer (yes/no) provide appropriate justification.

4) (8 pts) Let a function f: A → B be an injective function and g: B → C be a surjective function. With proof, determine if (a) g°f must be an injective function, and (b) g°f must be a surjective function. (If the answer is yes for a part, prove it in the general case. If the answer is no for a part, give a specific counter example where you completely specify A, B, C, f and g and explain how your specific example is a counter-example.)

COT 3100H Final Exam - Part C (Number Theory, Induction) - 30 pts

Date: 5/3/2021
Start Time: 11:20 am EDT
End Time: 12:00 pm EDT

Note: Since the answers to the number theory questions are not too hard to guess, ALL of the credit for those questions will be for the work and explanation. Either 0 or 1 point will be awarded for the actual answer. So be very careful to be thorough with your explanations on these questions.

1) (10 pts) Determine all integer solutions to the equation $504x + 150y = 24$.

2) (4 pts) Let k equal the maximum integer such that 500! is divisible by 40^k. What is k?

3) (8 pts) Define a recurrence relation as follows:

$a_0 = 1$, $a_1 = 3$
$a_n = 7a_{n-1} - 12a_{n-2}$, for all integers $n \geq 2$.

Make a guess for a closed form formula for a_n in terms of n and prove that guess via strong induction on n with 2 base cases. (Note: a guess would be something like $a_n = 2n^2 + 1$. Of course, this is not the correct answer! Just trying to show that the guess is to be some formula in terms of n which correctly describes the value of a_n recursively defined above.)

4) (8 pts) Prove for all non-negative integers n that $7 \mid (5^{2n+1} + 2^{2n+1})$

COT 3100H Final Exam - Part D (Counting, Probability) - 35 pts

Date: 5/3/2021
Start Time: 12:00 pm EDT
End Time: 12:50 pm EDT

1) (5 pts) A class has 10 girls and 15 boys. A quiz bowl team of 5 students from the class will be chosen such that at least 2 girls are on the team. In how many ways can the team be selected? Please leave your answer as a sum/difference/product of combinations.

2) (10 pts) A landscape architect intends on buying 50 plants. She will buy some hibiscus, bougainvillea, azalea, hydrangea and lilac. She will buy in between 5 and 20 of each variety. In how many different ways can she buy the plants?

3) (5 pts) Jessalyn has a lucky pair of shoes she wears to exams. Given that she is wearing her shoes, she gets an A on an exam 70% of the time. If she is not wearing her lucky pair of shoes, she gets an A only 40% of the time. Luckily she remembers to wear her lucky pair of shoes 95% of the time. On a particular exam, Jessalyn did not get an A. What is the probability she was wearing her lucky shoes on that day? **Express your answer as a fraction in lowest terms.**

4) (10 pts) Define a continuous random variable X as follows:

$$X = kx^2,\ 0 \le x \le 3$$
$$= 0, \text{ otherwise}$$

for a constant k.

(a) Find k.
(b) Find E(X) (expectation of X).
(c) Find Var(X). (Note: $Var(X) = E(X^2) - [E(X)]^2$. Also, translate the definition of E(X) and $E(X^2)$ from discrete random variables to continuous random variables. Namely, you are multiplying each probability by x for the first definition and by x^2 for the second and adding.)

5) (5 pts) In the game of Connect Four, how many of your pieces do you have to place in a row to win?