COT 3100 Recitation: Counting 2 Problems

Set #1

1) How many 15 letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters and no C's in the last 5 letters?

2) Call a number "prime-looking" if it is composite but not divisible by 2, 3 or 5. The three smallest prime-looking numbers are 49, 77 and 91. There are 168 prime numbers less than 1000. How many prime-looking numbers are less than 1000?

3) A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. For example, after traveling one mile the odometer changed from 000039 to 00050. If the odometer now reads 002005, how many miles has the car actually traveled?

4) Call a set of integers *spacy* if it contains no more than one out of any three consecutive integers. How many subsets of {1, 2, 3, …, 12}, including the empty set, are spacy?

Set #2

1) A line passes through A(1, 1) and B(100,1000). How many other points with integer coordinates are on the line and strictly between A and B?

2) An object in the plane moves from one lattice point to another. At each step, the object may move one unit to the right, one unit to the left, one unit up, or one unit down. If the object starts at the origin and takes a ten-step path, how many different points could be the final point?

3) How many ordered quadruplets (a_1, a_2, a_3, a_4) of non-negative integers, where at least one of the integers is even, satisfy the equation, $a_1 + a_2 + a_3 + a_4 = 100$?

4) How many non-negative integer solutions are there to the equation

$$a + b + c + d + e + f + g + h + i + j \leq 50?$$