<u>Spring 2021 Honors COT3100 Exam #2</u>

Date: 3/24/2021

Name: ____

1) (10 pts) Tubas weigh 39 pounds and trombones weigh 15 pounds. A high school has weighed all of its tubas and trombones and they weigh exactly 2313 pounds.

(a) Give one possible number of tubas and trombones (both non-negative integers) that satisfy the given information.

(b) With proof, determine the number of different ordered pairs of non-negative integers (x, y) satisfy the given information such that the school has x tubas and y trombones.

2) (10 pts) Let X equal the least common multiple of 840,000,000 and 99,000,000.

(a) Express X in prime factorized form.

(b) Find the number of divisors of X.

3) (10 pts) A fun trick to square a number ending in 5 is as follows:

1) Take the part preceding the digit 5 and call this part X.
2) Multiply X by (X+1) and write this down.
3) Add the digits 25 to the end of it.

For example, if we try to calculate 115^2, we get X = 11, and X(X+1) = 132, so the value of 115^2 = 13225, completing the trick.

Prove that this trick always works!

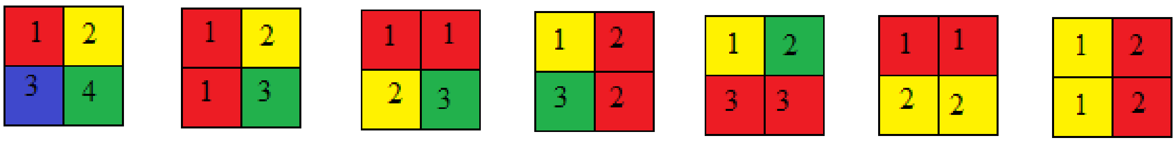
4) (12 pts) Recall that the Fibonacci numbers are defined as follows:

$F_0 = 0$, $F_1 = 1$, $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 2$.

Using induction on n, prove the following summation for all positive integers n:

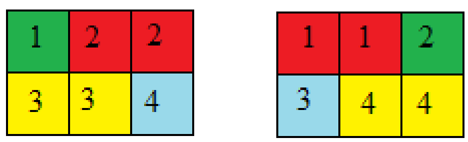
$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$$

5) (18 pts) Consider tiling a 2 x n hallway with tiles that are either size 1 x 1 or 1 x 2. Using strong induction on n with 2 base cases, prove that the tiling can be done in $\frac{3}{4}(3^n) + \frac{1}{4}(-1)^n$ ways, for all positive integers n. Two tilings are considered different if any square in one tiling is covered with either a different type of tile or a 1 x 2 tile in a different position between the two tilings. To provide some help, here are all the tilings for a 2 x 2 hallway:



Each individual tile is shown in a different color. For the printed version, since the colors don't show up well, each square that is part of the same tile is labeled with the same number. The first tiling has 4 separate 1 x 1 tiles. The following four tilings have exactly one 1 x 2 tile, oriented in 4 different ways. The last two tilings have two 1 x 2 tiles, both horizontal or both vertical.

The added restriction to the problem is that for all tiles that are horizontally placed, another horizontally placed tile is not allowed above or below it that is offset by exactly one square either way. Thus, for this problem, the two following tilings of a 2 x 3 hallway are not allowed:



Hint: To build all possible tilings of a 2 x (k+1) hallway, build off of all valid tilings of hallways of sizes 2 x k and 2 x (k - 1) respectively. This is only possible due to the restriction given.

In your solution, there's no need to reproduce the drawings above, just reference the drawings as needed.

6) (12 pts) Permutations - please leave your answers in factorials, combinations, powers, etc.

(a) How many permutations are there of the letters in the word CHATTANOOGA?

(b) How many of the permutations of the letters in CHATTANOOGA do NOT have any consecutive vowels?

(c) How many of the permutations of the letters in CHATTANOOGA contain the substring "HAT"?

7) (12 pts) How many solutions are there to the equation

$$a + b + c + d + e + f = 40$$

where a, b, c, d, e and f are non-negative integers that adhere to the following restrictions:

$$a \geq 3, \quad b \geq 4, \quad c \leq 12 \text{ and } e \geq 5?$$

8) (12 pts) Integers x and y with $x > y > 0$ satisfy $x + y + xy = (2^6)(3^7) - 1$. How many different ordered pairs (x, y) satisfy all of these requirements?

9) (4 pts) On what planet is the Ingenuity Mars helicopter currently located?

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Scratch Page - Please carefully mark any work on this page you would like graded.