COT 3100 Recitation: Random Algebra Problems Solutions

Set #1

1) The sum of Bob and Carol's age now is 100. Sixteen years ago, Bob was three times as old as Carol. How old was Bob when Carol was born?

Solution
B = Bob's current age
C = Carol's current age
B – C = Bob's age when Carol was born

B + C = 100
(B-16) = 3(C-16)

B = 100 – C, C = 100 – B
C = (B – 16)/3 + 16

B = 100 – [(B – 16)/3 + 16]
B = 100 – (B – 16)/3 - 16
B = 84 – (B – 16)/3
3B = 252 – (B – 16)
3B = 252 – B + 16
4B = 268
B = 67, currently
C = 100 – 67
C = 33, currently

B – C = 67 – 33
B – C = 34

2) If $\frac{x}{x-1} = \frac{y^2+2y-1}{y^2+2y-2}$, then what is x equal to, in terms of y?

Solution
We can divide both denominators into the numerators to yield:

$$1 + \frac{1}{x-1} = 1 + \frac{1}{y^2+2y-2}$$

$$\frac{1}{x-1} = \frac{1}{y^2+2y-2}$$

$$x - 1 = y^2 + 2y - 2$$

$$x = y^2 + 2y - 1$$

3) Three men, Bob, John and Steven, working together do a job in 6 hours less time than Bob alone, in 1 hour less time than John alone, and in one half the time needed by Steven, when working alone. How many hours would Bob and John working together take to do the job?

Solution

Let B finish b of the job in 1 hour, John finish j of the job in one hour and Steven finish s of the job in one hour. Let t be the amount of time it takes it takes to finish the job with all three working together. We have:

$$t(b+j+s)=1$$
$$(t+6)b=1$$
$$(t+1)j=1$$
$$2ts=1$$

Multiplying everything out and solving the last three for b, j and s we get:

$$tb+tj+ts=1$$
$$b=\frac{1}{t+6}, j=\frac{1}{t+1}, s=\frac{1}{2t}$$
$$\frac{t}{t+6}+\frac{t}{t+1}+\frac{t}{2t}=1$$
$$\frac{t}{t+6}+\frac{t}{t+1}+\frac{1}{2}=1$$
$$\frac{t}{t+6}+\frac{t}{t+1}=\frac{1}{2}$$

$$\frac{1}{t+6}+\frac{1}{t+1}=\frac{1}{2t}$$
$$\frac{(t+1)+(t+6)}{(t+1)(t+6)}=\frac{1}{2t}$$
$$2t(2t+7)=t^2+7t+6$$
$$4t^2+14t=t^2+7t+6$$
$$3t^2+7t-6=0$$
$$(3t-2)(t+3)=0$$

Since t is positive it follows that t = 2/3. So, with all three working together, they can finish the job in 40 minutes. It follows that b = $\frac{1}{{}^2/_3+6}=\frac{3}{20}$ and j = $\frac{1}{{}^2/_3+1}=\frac{3}{5}$. We want to solve for t" in the equation:

$$t''\left(\frac{3}{20}+\frac{3}{5}\right)=1$$
$$t''\left(\frac{15}{20}\right)=1$$
$$t''=\frac{4}{3}$$

Together Bob and John could finish the job in 1 and 1/3 hours.

4) It is now in between 10:00 and 11:00 o'clock, and six minutes from now, the minute hand of the watch will be exactly opposite the place where the hour hand was three minutes ago. What is the exact time now?

Solution

We know that at 10:00, the angle of the minute hand is $M = 0°$, the hour hand is $H = 300°$.

We also know that the minute hand moves 6° each minute, and that the hour hand moves 0.5° each minute.

So, the angles can be expressed as follows, where m is the minutes passed since 10 o'clock:

$$M = 6m$$
$$H = 300 + \frac{m}{2}$$

Then, to account for the change in minutes and angle, we can set up the equation and solve:

$$6(m + 6) + 180 = 300 + \frac{(m-3)}{2}$$
$$6m + 216 = \frac{600+m-3}{2}$$
$$12m + 432 = 597 + m$$
$$11m = 165$$
$$m = 15$$

It follows that the exact time now is **10:15 AM.**

Set #2

1) A teen age boy wrote his own age after his father's. (Both are in between 10 and 99, inclusive.) From this new 4 digit number he subtracted the absolute value of the difference of their ages to get 4289. What is the sum of their ages?

Solution

Since his age is less than 100, it follows that the father's age is 43. (Since the boy is a teenager, he is less than 20 and the difference of their ages is at least 22.) Let x be the age of the teenager. They we have:

$$4300 + x - (43 - x) = 4289$$
$$4300 + x - 43 + x = 4289$$
$$4257 + 2x = 4289$$
$$2x = 32$$
$$x = 16$$

Thus, the boy is 16 and his father is 43. **The sum of their ages is 59.**

2) When one ounce of water is added to a mixture of acid and water, the resulting mixture is 20% acid. When one ounce of acid is added to this new mixture, the resulting mixture is 1/3 acid. What is the percentage of acid in the original mixture?

Solution

Let the original mixture have a ounces of acid and x ounces total. Using the given information, we have:

$$\frac{a}{x+1} = \frac{1}{5} \qquad \text{and} \qquad \frac{a+1}{x+2} = \frac{1}{3}$$

Cross multiplying, we find $5a = x + 1$ and $3a + 3 = x + 2$. Solving for x in both equations yields:

$$x = 5a - 1 = 3a + 1$$

It follows that $a = 1$ and $x = 4$. The original mixture was **25%** acid.

3) Al gets the disease algebritis and must take one green pill and one pink pill each day for two weeks. A green pill costs $1 more than a pink pill and Al's pills cost a total of $546 for the two weeks. How must does one green pill cost?

Solution

Let the green pill cost g dollars. Then the pink pill costs g - 1 dollars. Thus we have:

$$14(g + g - 1) = 546$$
$$2g - 1 = 39$$
$$g = 20$$

One green pill costs **$20.**

4) Cassandra sets her watch to the correct time at noon. At the actual time of 1:00 PM, she notices that her watch reads 12:57 and 36 seconds. Assuming that her watch loses time at a constant rate, what will be the actual time when her watch first reads 10:00 PM?

Solution

There are 3600 seconds in an hour. Cassandra's watch ticks $57 \times 60 + 36 = 3456$ seconds in this time. Thus, the actual time moves at a rate of $\frac{3600}{3456} = \frac{225}{216}$ compared to Cassandra's clock. Thus, the number of actual seconds that elapse by the time Cassandra's clock reads 10:00 PM (when it had ticked $10\ hr \times \frac{3600 ticks}{hr} = 36000 ticks$, can be calculated as follows:

$$\frac{225 sec}{216 ticks} \times 36000 ticks = 225 \times \frac{1000}{6} sec \times \frac{1 hr}{3600 sec} = 10\frac{5}{12} hr$$

Since there are 60 minutes in an hour, this equates to **10:25 PM** for the actual time.