Spring 2021 Honors COT3100 Exam #1

Date: 2/8/2021

1) (10 pts) Complete the following truth table. Please write T for true and F for false.

p	q	r	$\overline{p \vee r}$	$\bar{q} \wedge r$	$\overline{p \vee r} \vee (\bar{q} \wedge r)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (15 pts) Using the laws of logic, show that the following logical expression is a tautology:

$$((q \rightarrow (p \wedge r)) \wedge \bar{p}) \rightarrow \bar{q}$$

Please use the format shown in class to express your answer.

3) (10 pts) Use the rules of implication to make the following argument:

$$p \rightarrow q$$
$$r \rightarrow s$$
$$\bar{q} \vee \bar{s}$$
$$t \rightarrow p$$
$$u \rightarrow r$$
$$u$$

$$\therefore \bar{t}$$

Step	**Reason**
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	

Note: It's unlikely you will use all the rows provided.

4) (10 pts) Prove or disprove the following assertion for all sets A, B, C and D:

$$(A \cup B) \times (C \cap D) \subseteq (A \times D) \cup (B \times C)$$

5) (10 pts) Prove or disprove the following assertion for all sets A and B:

$$if\, A - B \subseteq C - D, then\, A \subseteq (C \cup D)$$

6) (10 pts) In the string orchestra, each student plays at least one of the following three instruments: violin, viola, and cello. The number of students who play the violin, plus the number of students who play the viola, plus the number of students who play the cello is 45. (Note that a student who plays two of the three instruments is counted twice in this count and a student who plays all 3 is counted 3 times in this count.) The number of students who play either the violin or viola or both is 17, and the number of students who play all three instruments is 5. How many students are in either (or both) of the following groups?

(a) students who play both the violin and viola
(b) students who play the cello

7) (10 pts) Let r and s be the roots of the equation $3x^2 - 12x + 5 = 0$. What is the quadratic equation with leading coefficient of 1 with the roots r^3 and s^3? **(Note: Of the two coefficients not equal to 1, one will be a non-integer fraction and the other one will be an integer.)**

8) (10 pts) A river runs East-West with the current flowing west. Kayla starts canoeing from her house, going east, against the current. After 80 minutes, Kayla reached a nice waterfall and decided to turn around and starts canoeing west. She passes her house and eventually stops after another 160 minutes. It turns out that when she finished the trip, she has twice as far from her house as she was when she was at the waterfall. If the entire trip was 16 miles long, what is her rate (in miles per hour) of rowing in still water, and what is the rate of the river current (in miles per hour)?

9) (10 pts) Let x, y and z satisfy the following equations:

$$4^x = 8,\ 8^y = 32,\ y - x = log_{64} z$$

What are the values of x, y and z? Express all answers as fractions in lowest terms or integers.

10) (5 pts) What color does the commonly used food dye Yellow 5 make food?

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Scratch Page - Please clearly mark any work on this page you would like graded.