

COT 3100 Recitation: Logs - Solutions for Set #1

1) What is the value of $(81^{\log_3 1234})^{0.25}$?

$$\begin{aligned} & (3^{4\log_3 1234})^{0.25} \\ & 3^{4\log_3 1234(0.25)} \\ & 3^{\log_3 1234} = 1234 \end{aligned}$$

Recall that $(a^b)^c = a^{bc}$, this says multiply a by itself b times, then take that number and multiply it by itself c times: $a^b a^b \dots a^b$ (c times). How many times is a multiplied by itself? Bc.

2) Determine the ordered pair, (a, b), that satisfies the following pair of equations:

$$\log_{16} a^2 + \log_8 b^3 = 11$$

$$\log_8 a^6 + \log_{16} b^{10} = 32$$

Note: You may express a and b as a some base raised to an exponent instead of a single value, if you wish.

$$\begin{aligned} 2\log_{16} a + 3\log_8 b &= 11 \\ \frac{2\log_2 a}{\log_2 16} + \frac{3\log_2 b}{\log_2 8} &= 11 \\ \frac{2\log_2 a}{4} + \frac{3\log_2 b}{3} &= 11 \end{aligned}$$

$$6\log_8 a + 10\log_{16} b = 32$$

$$\begin{aligned} \frac{6\log_2 a}{3} + \frac{10\log_2 b}{4} &= 32 \\ \frac{2\log_2 a}{4} + \frac{3\log_2 b}{3} &= 11 \end{aligned}$$

Let $X = \log_2 a$, $Y = \log_2 b$

$$2x + \frac{5}{2}y = 32$$

$$2x + 4y = 44$$

Subtract second from the first: $-\frac{3}{2}y = -12$, $y = 8$, $x = 6$

Finally $a = 2^6 = 64$, $b = 2^8 = 256$

3) What is the value of a for which $\sum_{i=2}^{10} \frac{1}{\log_i a} = 1$?

$$\frac{1}{\log_2 a} + \frac{1}{\log_3 a} + \frac{1}{\log_4 a} + \dots + \frac{1}{\log_{10} a} = 1$$

$$\frac{1}{\frac{\ln a}{\ln 2}} + \frac{1}{\frac{\ln a}{\ln 3}} + \frac{1}{\frac{\ln a}{\ln 4}} + \dots + \frac{1}{\frac{\ln a}{\ln 10}} = 1$$

$$\frac{\ln 2}{\ln a} + \frac{\ln 3}{\ln a} + \frac{\ln 4}{\ln a} + \dots + \frac{\ln 10}{\ln a} = 1$$

$$\log_a 2 + \log_a 3 + \log_a 4 + \dots + \log_a 10 = 1$$

$$\log_a 10! = 1$$

So $a = 10!$

4) The sequence $\log_{12} 162, \log_{12} x, \log_{12} y, \log_{12} z, \dots, \log_{12} 1250$ is an arithmetic progression. What is x ?

Let $D = \log_{12} x - \log_{12} 162$, the common difference of the arithmetic progression.

$$\log_{12} 162 + 4D = \log_{12} 1250$$

$$4D = \log_{12} 1250 - \log_{12} 162$$

$$4D = \log_{12} (1250/162) = \log_{12} (625/81) = \log_{12} (5^4/3^4)$$

$$4D = \log_{12} (5/3)^4$$

$$4D = 4\log_{12} (5/3)$$

$$D = \log_{12} (5/3)$$

$$\log_{12} 162 + \log_{12} (5/3) = \log_{12} x$$

$$\log_{12} 270 = \log_{12} x, \text{ so } x = 270.$$

COT 3100 Recitation : Logs - Solutions for Set #2

Problems For Recitation

1) Solve for x in the following equation: $\log_2 x^3 + \log_4(8x^2) = -\frac{5}{2}$

Solution

$$\log_2 x^3 + \log_4(8x^2) = -\frac{5}{2}$$

$$3\log_2 x + \log_4 8 + \log_4 x^2 = -\frac{5}{2}$$

$$3\log_2 x + \frac{3}{2} + 2\log_4 x = -\frac{5}{2}$$

$$3\log_2 x + 2\left(\frac{\log_2 x}{\log_2 4}\right) = -4$$

$$3\log_2 x + 2\left(\frac{\log_2 x}{2}\right) = -4$$

$$4\log_2 x = -4$$

$$\log_2 x = -1$$

$$\text{So, } x = 2^{-1} = \frac{1}{2}.$$

Note: We can evaluate $\log_4 8$, by solving $4^x = 8$, so $2^{2x} = 2^3$, since the exponential function is one to one, we know that $2x = 3$, so $x = \frac{3}{2}$.

2) If $3^{\log_9 x} = 9^{\log_3 27}$, what is the prime factorization of x?

Solution

$$3^{\log_9 x} = 9^{\log_3 27}$$

$$3^{\log_9 x} = 9^3$$

$$3^{\log_9 x} = (3^2)^3$$

$$3^{\log_9 x} = 3^6$$

Since the exponential function is one to one, we know that:

$$\log_9 x = 6$$

It follows that $x = 9^6 = (3^2)^6 = \mathbf{3^{12}}$, in prime factorized form.

3) What is the value of the following summation?

$$\sum_{i=1}^n \log_{n!} i$$

Solution

$$\sum_{i=1}^n \log_{n!} i = \log_{n!} \left(\prod_{i=1}^n i \right) = \log_{n!} n! = 1$$

Note: The sigma means to add the terms $\log 1 + \log 2 + \log 3 + \dots + \log n$, where the base is $n!$. The log addition rule says that to put these in a single log, we multiply each item we are taking the log of. The pi sign means exactly that, to multiply each term you get when you plug i into the expression on the inside and the product of each integer from 1 to n is defined as n factorial ($n!$).

4) Define a function f as follows: $f(1) = 2$. For all integers $n > 1$, $f(n) = (f(n - 1))^{2^n}$.

What is the value of $\log_{65536}(\log_{65536}(f(4)))$?

Express your answer in the form $2^a + 2^b$, where both a and b are integers. Note that $65536 = 2^{16}$.

Solution

First, let's calculate $f(4)$. $f(2) = f(1)^4 = 2^4$, $f(3) = (f(2))^8 = (2^4)^8 = 2^{32}$, $f(4) = f(3)^{16} = (2^{32})^{16} = 2^{512}$.

Let $X = \log_{65536}(f(4))$. Then, using the power rule, we have $X = 512 \log_{65536} 2$. Use the log base change rule with base 2 to get the following:

$$\log_{65536} 2 = \frac{\log_2 2}{\log_2 65536} = \frac{1}{16}$$

Thus, $X = \frac{512}{16} = 32$.

The value we desire is $\log_{65536} X = \log_{65536} 32 = \frac{\log_2 32}{\log_2 65536} = \frac{5}{16} = 2^{-2} + 2^{-4}$.