**COT 3100 Recitation: D = RT - Solutions for Recitation Problems Set #1**

1) Jenny drives from home to the beach, averaging 30 miles per hour. On the return trip, she averages 60 miles an hour. What is the average speed for her round trip?

**Solution**

Let D be the distance from home to the beach, in miles.
The time it takes to get to the beach is $\frac{D}{30}$ hours.
The time it takes to go from the beach to home is $\frac{D}{60}$ hours.
Thus, the round trip time is $\frac{D}{30} + \frac{D}{60}$ hours.
The round trip distance is 2D. It follows we can solve for the average speed of the trip by taking this distance and dividing by the time of the total trip:

$$r = \frac{2D}{\frac{D}{30} + \frac{D}{60}} = \frac{2}{\frac{1}{30} + \frac{1}{60}} = \frac{2}{\frac{2+1}{60}} = \frac{120}{3} = \boldsymbol{40\ mph}$$

2) Mr. Earl E. Bird leaves his house for work at exactly 8 AM every morning. When he averages 40 miles per hour, he arrives at his workplace three minutes late. When he averages 60 miles per hour, he arrives three minutes early. At what average speed, in miles per hour, should Mr. Bird drive to arrive at his workplace precisely on time?

**Solution**

Let's change all the time units to hours, so 3 minutes = $\frac{1}{20}$ hours. Let t be the amount of time in hours his trip should take and let D be the distance between his house and work. Using the information given, we have:

$$D = 40\left(t + \frac{1}{20}\right) = 60(t - \frac{1}{20})$$

Simplifying, we have $40t + 2 = 60t - 3$, which leads to $20t = 5$. It follows that the amount of time his trip should have taken is $\frac{5}{20} = \frac{1}{4}$ hours. Plug this into either equation above to find that $D = 60\left(\frac{1}{4} - \frac{1}{20}\right) = 60 \times \frac{1}{5} = 12$ miles.

To answer the question posed, we want to figure out at what rate to drive to traverse 12 miles in exactly $\frac{1}{4}$ hours. $r = \frac{12 miles}{1/_4 hrs} = \boldsymbol{48mph}$

3) David drives from his home to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. How many miles is the airport from his home?

**Solution**

Let t be the amount of time the entire trip *should* take. He drives 35 mph for 1 hour, and then drives 50 mph for $t - \frac{3}{2}$ hours. (This is because for one hour he wasn't driving 50 mph and finishes his trip a ½ hour early, so we have to subtract both 1 and ½ from t.) Let D be the distance to the airport. Since he'll be one hour late if he continues the whole trip at 35 mph, we have $D = 35(t + 1)$.

Using the actual trip information, we find that $D = 35 + 50(t - \frac{3}{2})$, since he went 35 miles the first hour and $50(t - \frac{3}{2})$ for the rest of the trip. Set these two expressions for D equal to each other:

$$35(t + 1) = 35 + 50(t - \frac{3}{2})$$
$$35t + 35 = 35 + 50t - 75$$
$$15t = 75$$
$$t = 5$$

Now, solve for D: $D = 35(5 + 1) = 35 \times 6 = \mathbf{210}$ **miles.**

4) A swimming pool is the shape of a rectangular prism with a length of 12 feet, a width of 10 feet and a depth of 8 feet. The pool is full at Tuesday at 8 am but springs a leak from the bottom of the pool surface that leaks 1 cubic inch of water per second into the ground. (This means that slowly, the water level in the pool decreases.) How much lower (in inches) is the water level at Wednesday morning at 8 am as compared to Tuesday at 8 am when the pool was full? Which piece of information given in the problem is mostly irrelevant?

**Solution**

The area of the surface of the pool is 12 ft x 10 ft = 144 in x 120 in, since there are 12 inches in a foot. So, the water level decreases by 1 inch for every 144 x 120 cubic inches that leak. The time period in question is one full day (24 hours) exactly. There are 24 x 60 x 60 seconds in a full day, since there are 24 hours in a day, 60 minutes in an hour and 60 seconds in a minute.

It follows that the pool loses 24 x 60 x 60 cubic inches of water. To determine how much lower the water level will be in inches, divide this value by the cross-sectional area of the pool. Notice the unit matching!

Loss of water level = $\frac{24\times60\times60\ in^3}{144\times120\ in^2} = \frac{60\times60}{6\times120} in = \frac{10}{2} in = \mathbf{5\ \mathit{inches}}$

The depth of the pool is largely irrelevant, as long as the water doesn't get completely depleted. Alternatively, one could say that knowing the pool is full is unnecessary. Another accepted answer was knowing exactly where the leak was. (The pool just have to have enough water not to empty out and the leak just has to be below the level that gets drained.)

**COT 3100 Recitation: D = RT - Solutions for Recitation Problems Set #2**

1) Casey drove a total of 100 miles. For the first portion of the trip she averaged 40 miles per hour and for the second/last portion of the trip she averaged 55 miles per hour. If her average speed for the entire trip was 50 miles an hour, how long (in miles) was the first portion of her trip? Please express your answer as a fraction in lowest terms.

**Solution**

Casey drove a total of 100 miles. For the first portion of the trip she averaged 40 miles per hour and for the second/last portion of the trip she averaged 55 miles per hour. If her average speed for the entire trip was 50 miles an hour, how long (in miles) was the first portion of her trip? Please express your answer as a fraction in lowest terms.

Although the distance of the first portion of the trip is unknown, lots of other stuff is known. It's immediately clear that we should let d = the distance of the first portion of the trip and that the second portion of the trip has distance 100 - d. Now, we can let t1 = the time for the first portion of the trip and t2 = time for the second portion of the trip and t3 = time for the full trip. Since we know the average speed and distance of the whole trip, we get the following equations for the two portions of the trip and the whole trip itself:

d = 40(t1)
100 - d = 55(t2)
100 = 50 (t3)

Also, by definition, we have:

t3 = t2 + t1

Using the last equation, we get t3 = 2. We can also solve for t2 and t1 as follows:

$$t1 = \frac{d}{40}$$

$$t2 = \frac{100-d}{55}$$

Now, just plug into that last equation to get:

$$2 = \frac{d}{40} + \frac{100-d}{55}$$

From here, it’s just algebra for one equation with one variable. Get a common denominator first:

$$2 = \frac{11d}{40(11)} + \frac{8(100-d)}{(8)55}$$

Now, multiply by that denominator:

$$2(40)(11) = 11d + 800 - 8d$$

$(80)(11) = 3d + 80(10)$
$3d = 80(11) - 80(10)$
$3d = 80(11 - 10)$
$3d = 80$
$d = \mathbf{\frac{80}{3}}$

So, the key here was identifying three trips (portion1, portion2, whole) and the associated variables. Then, creating each of the relevant equations and attempting to reduce the number of variables as fast as possible. This left us with one equation in one variable, which is ideal. Also, notice some of the algebra above - multiplications were not done. Instead expressions were factored in the hope that stuff would drop out, which it did! This is key for exams w/o calculators.

2) Jessica is going from Orlando to Miami and she takes a 15 minute break at Vero Beach. Her goal is to average 60 miles per hour for the whole trip. The distance between Orlando and Vero Beach is 100 miles and the distance between Vero Beach and Miami is 140 miles. If her average driving speed from Orlando to Vero Beach is 50 miles per hour, how fast must her average speed be driving from Vero Beach to Miami to achieve her goal?

**Solution**
The total distance to drive is 240 miles and if we aim to achieve an average of 60 miles per hour, we must make the drive in 240 miles/60 mph = 4 hours.

If we drive 100 miles averaging 50 miles per hour, we take 100 miles/50 mph = 2 hours.

Then we take a break for 15 minutes. This means we've used 2 hours and 15 minutes.

We have 4 hours - (2 hours 15 minutes) = 1 hour 45 minutes = $\frac{7}{4}$ hours to complete the trip.

Let r be the average speed we drive from Vero Beach to Orlando. Then we have:

$$140\ miles = r(\frac{7}{4} hours)$$
$$r = 140 \times \frac{4}{7} mph = \mathbf{80\ mph}$$

3) The current in a river is flowing steadily at 3 miles per hour. A motor boat which travels at a constant rate in still water goes downstream 4 miles and then returns to its starting point. The trip takes one hour, excluding the time spent in turning the board around. What is the ratio of the downstream to the upstream rate?

**<u>Solution</u>**

Let the rate of the motor boat in still water be r miles per hour. In total the motor boat travels 8 miles, traveling 4 of those miles at (r+3) mph and traveling the other 4 miles at (r - 3) mph. The question asks us to find the ration (r+3) to (r-3). Our d = rt equation when equating time is:

$$1 = \frac{4}{r+3} + \frac{4}{r-3}$$

$$1 = \frac{4(r-3)}{(r+3)(r-3)} + \frac{4(r+3)}{(r+3)(r-3)}$$

$$(r+3)(r-3) = 4r - 12 + 4r + 12$$

$$r^2 - 9 = 8r$$

$$r^2 - 8r - 9 = 0$$

$$(r-9)(r+1) = 0$$

$r = 9$, since r is positive.

It follows that the desired ratio is (9+3):(9-3) or **2: 1.**

4) Selena and Trina live 13 miles apart. Yesterday Selena started to ride her bicycle toward Trina's house. A little later Trina started to ride her bicycle towards Selena's house. When they met, Selena had ridden for twice the length of time as Trina and at four-fifths of Trina's rate. How many miles had Trina ridden when they met?

**<u>Solution</u>**

Let Trina's rate be *r* and the amount of time Trina rides the bicycle be *t*. Selena's rate is $\frac{4}{5}r$ and the amount of time she rode her bicycle is *2t*. Summing the distances they rode we get:

$$13 = rt + \frac{4}{5}r(2t)$$

$$13 = \frac{13}{5}rt$$

$$rt = 5$$

Since the distance Trina rode is *rt*, the desired answer is **<u>5 miles.</u>**