COT 3100 Fall 2022 Homework #8
Please Consult WebCourses for the due date/time

Note: Please *justify* your answers and why you use each formula. Please leave answers in factorials, combinations and powers.

1) (10 pts) How many solutions does the equation a + b + c + d + e = 45 have, where a, b, c, d and e are each non-negative integers, under the following restrictions?

a) No other restrictions

b) $d \geq 5$

c) $b \leq 10$

d) $b \leq 10$ and $c \leq 8$

e) $d \geq 5$ and $b \leq 10$ and $c \leq 8$

2) (5 pts) How many solutions does the equation $a + b + c + d + e + f + g + h + i + j \leq 55$ have if each variable must be a non-negative integer?

3) (10 pts) How many ordered quadruplets (a_1, a_2, a_3, a_4) of non-negative integers, where at least one of the integers is even, satisfy the equation $a_1 + a_2 + a_3 + a_4 = 100$? Please express your answer in the form $\binom{w}{x} - \binom{y}{z}$. (Note that the values of w, x, y and z will be integers, but not necessarily all distinct.)

4) (5 pts) Suppose that one person in 1,000 people has a rare genetic disease. There is an excellent test for the disease; 97% of the people with the disease test positive and only 2% of the people who don't have it test positive. What is the probability that someone who tests positive has the disease? What is the probability that someone who tests negative does not have the disease?

5) (5 pts) A partial deck of Uno cards has 20 red cards and 20 blue cards. You draw, without replacement, 5 cards from the deck. The "most equal" split of colors you can receive is 2 cards of one color and three cards of the other color. What is the probability that you'll receive this "most equal" split of colors?

6) (5 pts) The integers from 1 to 15, inclusive, are partitioned at random into two sets, one with 7 elements and the other with 8. What is the probability that 1 and 2 are in the same set?

7) (5 pts) In a dice game, a player rolls 6 fair standard six sided dice. The player gets some points for all of the following:

a) Three **or more** (of a kind, so three or more 4's for example)
b) Three pairs (so two 3s, two 4s and two 6s for example)
c) All six distinct numbers (so in any order 1, 2, 3, 4, 5, 6)
d) Rolling the number 1 or the number 5

What is the probability, when rolling the six dice, that a player gets no points?

8) (5 pts) Give a summary of the life and mathematical contributions of Evariste Galois. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.